Transverse Momentum Parton Distributions and Gauge Links

31 May 2010

The College of William and Mary

Leonard Gamberg
Penn State University

Outline

- **Transverse spin Effects in TSSAs**
- **Gauge links-Color Gauge Inv.-“T-odd” TMDs**
- **T-odd PDFs via FSIs & “Transverse distortion”**

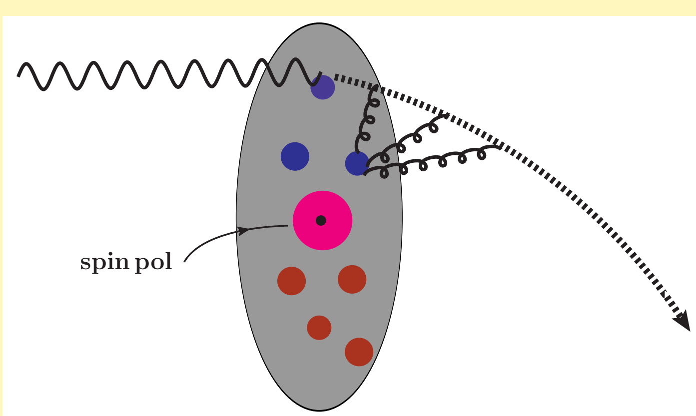
“QCD calc“ FSIs Gauge Links-Color Gauge Inv.“T-odd” TMDs

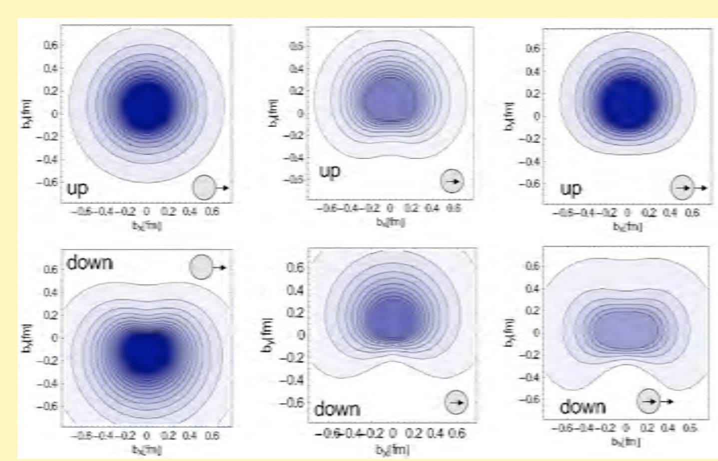
“Pheno” -Transverse Structure TMDs and TSSAs-**b** and **k** asymm

An improved dynamical approach for FSIs & model building

$f_{1T}^{\perp}(x, \mathbf{k}_{\perp}^{2})$

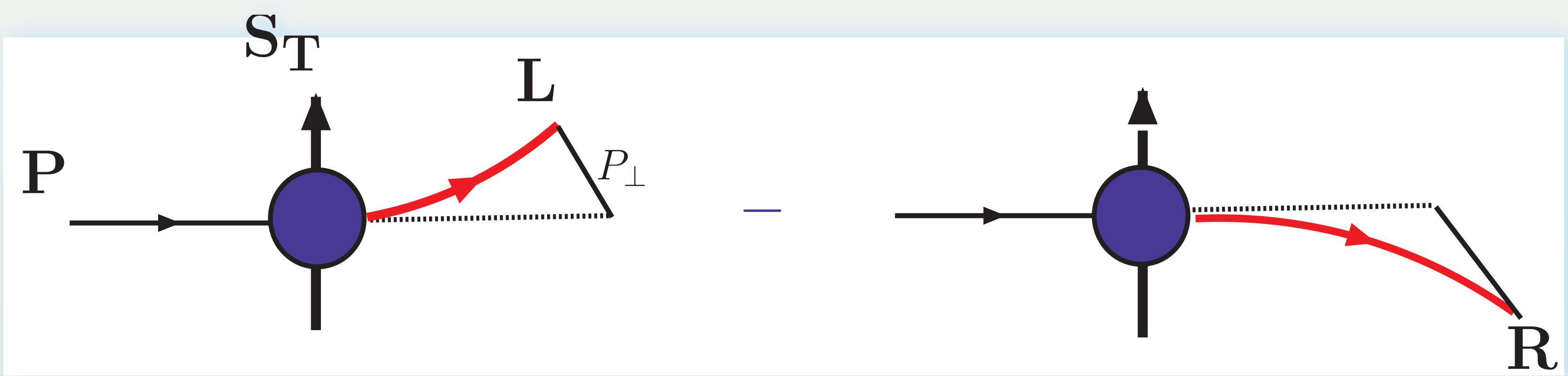
$\mathcal{E}(x, \mathbf{b}_{\perp}^{2})$





Transverse SPIN Observables SSA (TSSA) $P^{\uparrow}P \to \pi X$

- **Single Spin Asymmetry**



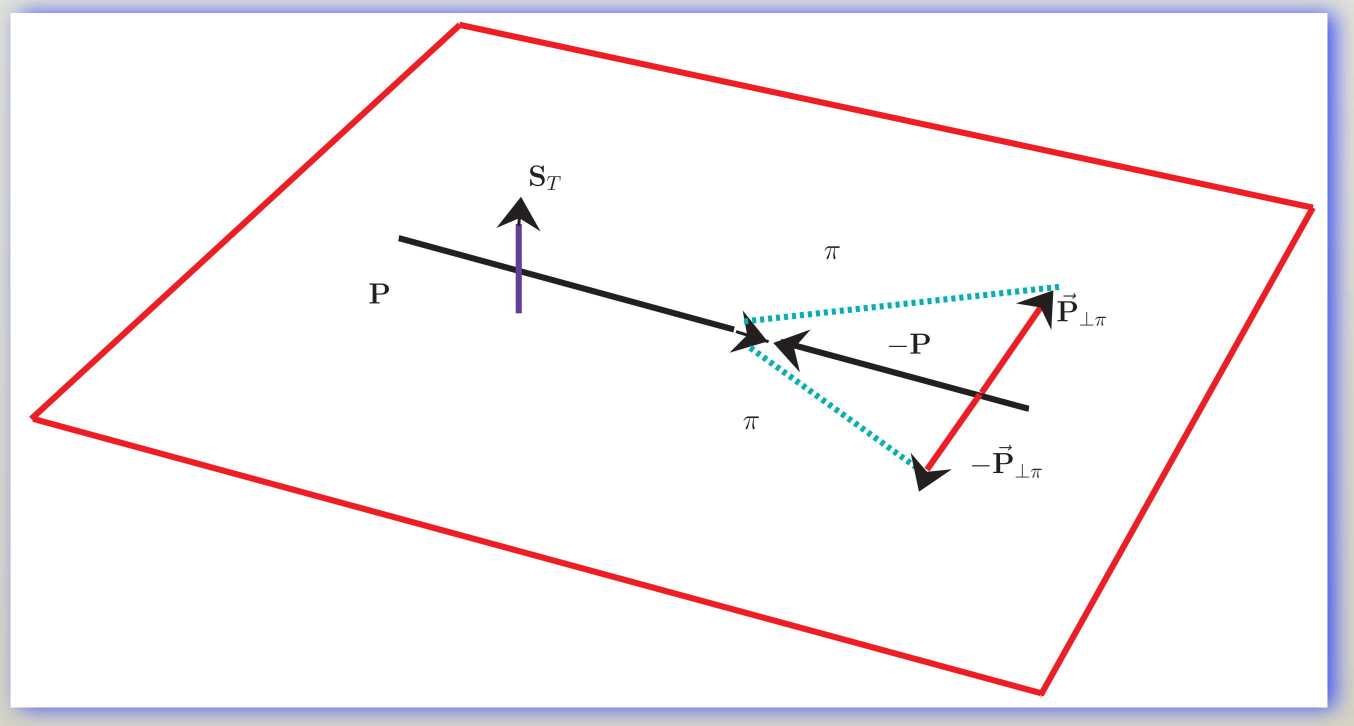
Parity Conserving interactions: SSAs Transverse Scattering plane

$$\Delta\sigma \sim i S_T \cdot (\mathbf{P} \times P_\perp^\pi)$$

- **Rotational invariance** $\sigma^{\downarrow}(x_F, \boldsymbol{p}_\perp) = \sigma^{\uparrow}(x_F, -\boldsymbol{p}_\perp)$

  $\Rightarrow$ ***Left-Right Asymmetry***

$$\boldsymbol{A}_N = \frac{\sigma^{\uparrow}(x_F,\boldsymbol{p}_\perp) - \sigma^{\uparrow}(x_F,-\boldsymbol{p}_\perp)}{\sigma^{\uparrow}(x_F,\boldsymbol{p}_\perp) + \sigma^{\uparrow}(x_F,-\boldsymbol{p}_\perp)} \equiv \Delta\sigma$$



Reaction Mechanism

⋆ **Co-linear factorized QCD-parton dynamics**

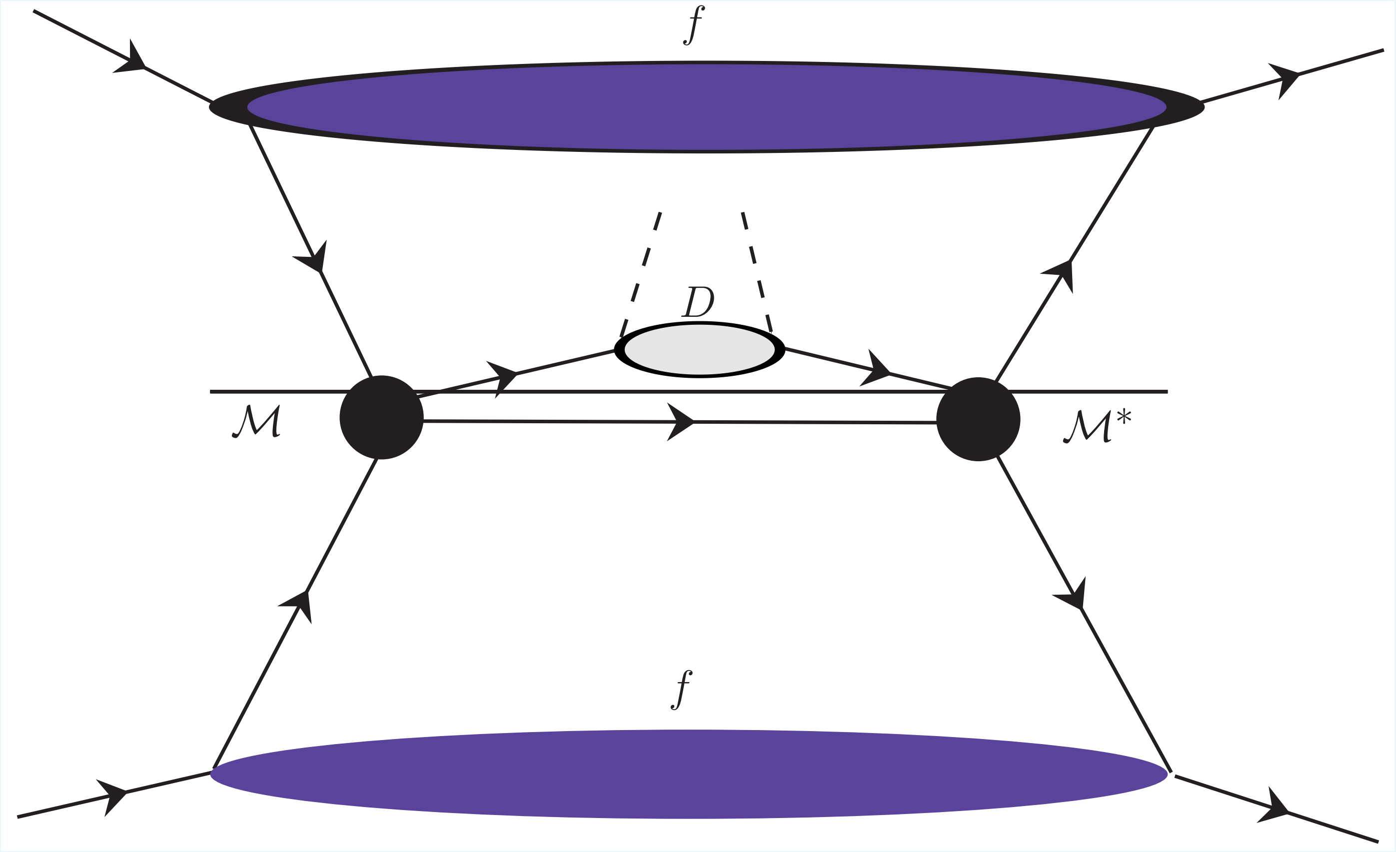
$\Delta\sigma^{pp^\uparrow \to \pi X} \sim f_a \otimes f_b \otimes \Delta\hat{\sigma} \otimes D^{q\to\pi}$

Requires helicity flip-hard part $\Delta\hat{\sigma} \equiv \hat{\sigma}^\uparrow - \hat{\sigma}^\downarrow$

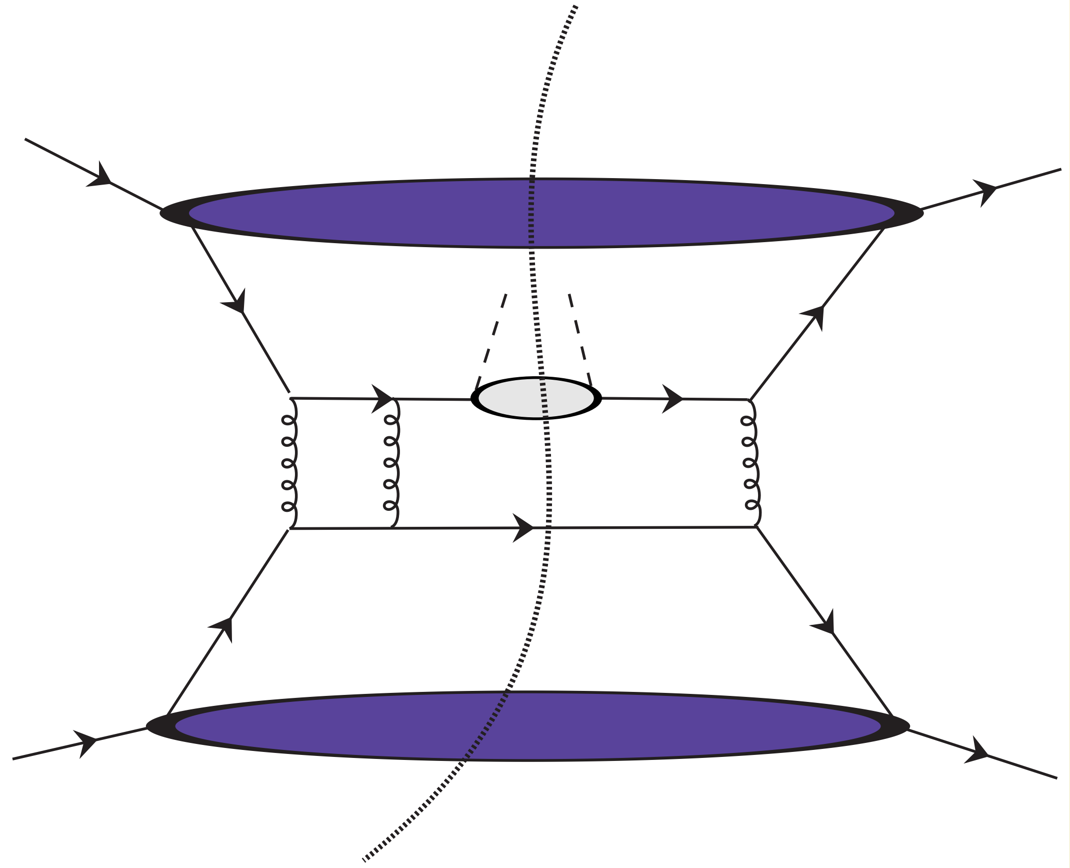
⋆ **TSSA requires relative phase btwn *different* helicity amps**

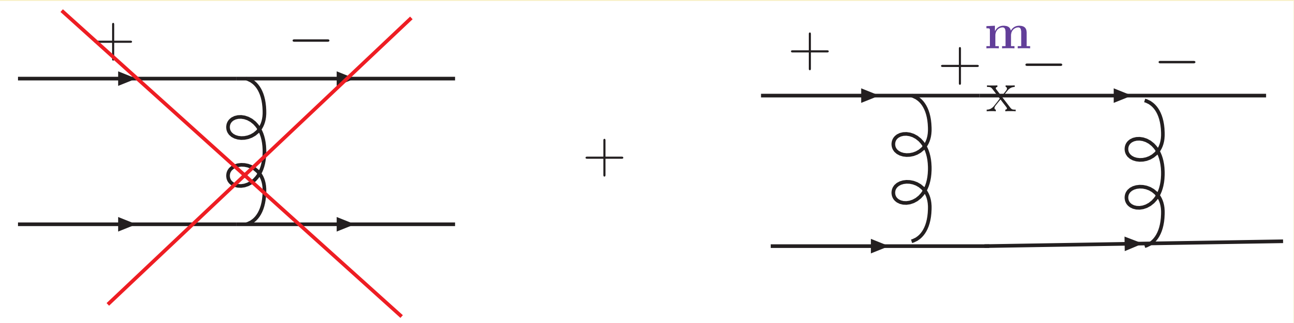
$$\hat{a}_N = \frac{\hat{\sigma}^\uparrow - \hat{\sigma}^\downarrow}{\hat{\sigma}^\uparrow + \hat{\sigma}^\downarrow} \sim \frac{\mathrm{Im}\left(\mathcal{M}^{+*}\mathcal{M}^{-}\right)}{|\mathcal{M}^+|^2 + |\mathcal{M}^-|^2}$$

$$|\uparrow / \downarrow\rangle = (|+\rangle \pm i|-\rangle)$$



Factorization Theorem in QCD Helicity limit....triviality.....





- **QCD interactions conserve helicity**
  $m_q \to 0$ **and Born amplitudes real**

$\star$ $A_N \sim \frac{\boldsymbol{m}_q \alpha_s}{E}$ **Kane, Repko, PRL:1978**

Twist three and trival?!

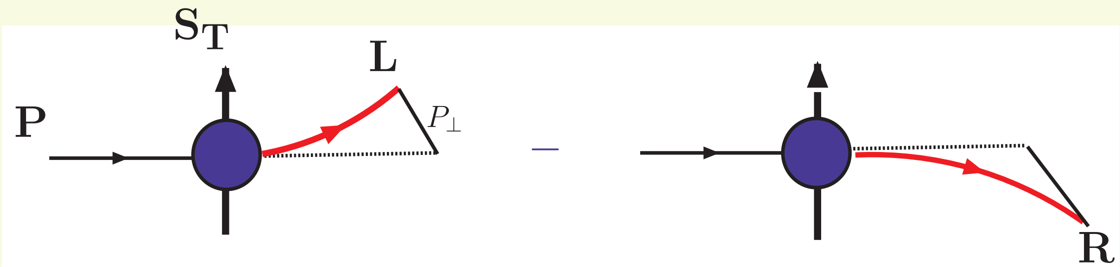
Not the full story @ Twist 3 approach ETQS approach

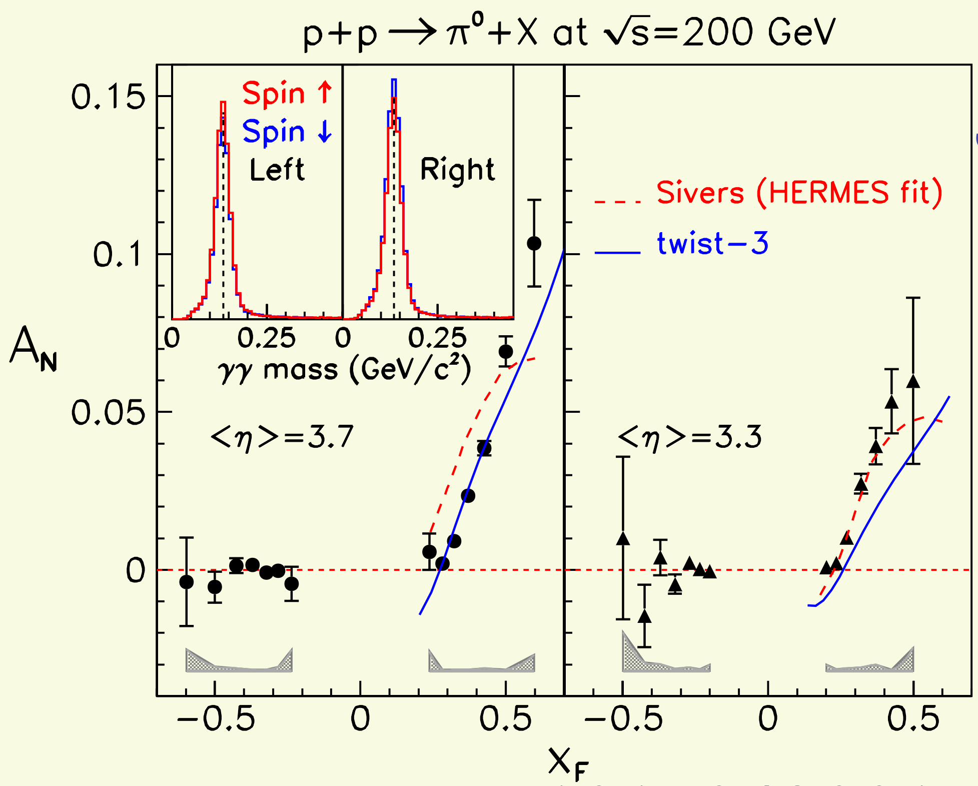
Phases in *soft* poles of propagator in hard subprocess Efremov & Teryaev :PLB 1982

Qiu-Sterman:PLB 1991, 1999, Koike et al. PLB 2000. . . 2007,

Ji,Qiu,Vogelsang,Yuan:PR 2006,2007. . .

Large Transverse SSA's at $\sqrt{s}$ = 62.4 & 200 GeV at RHIC





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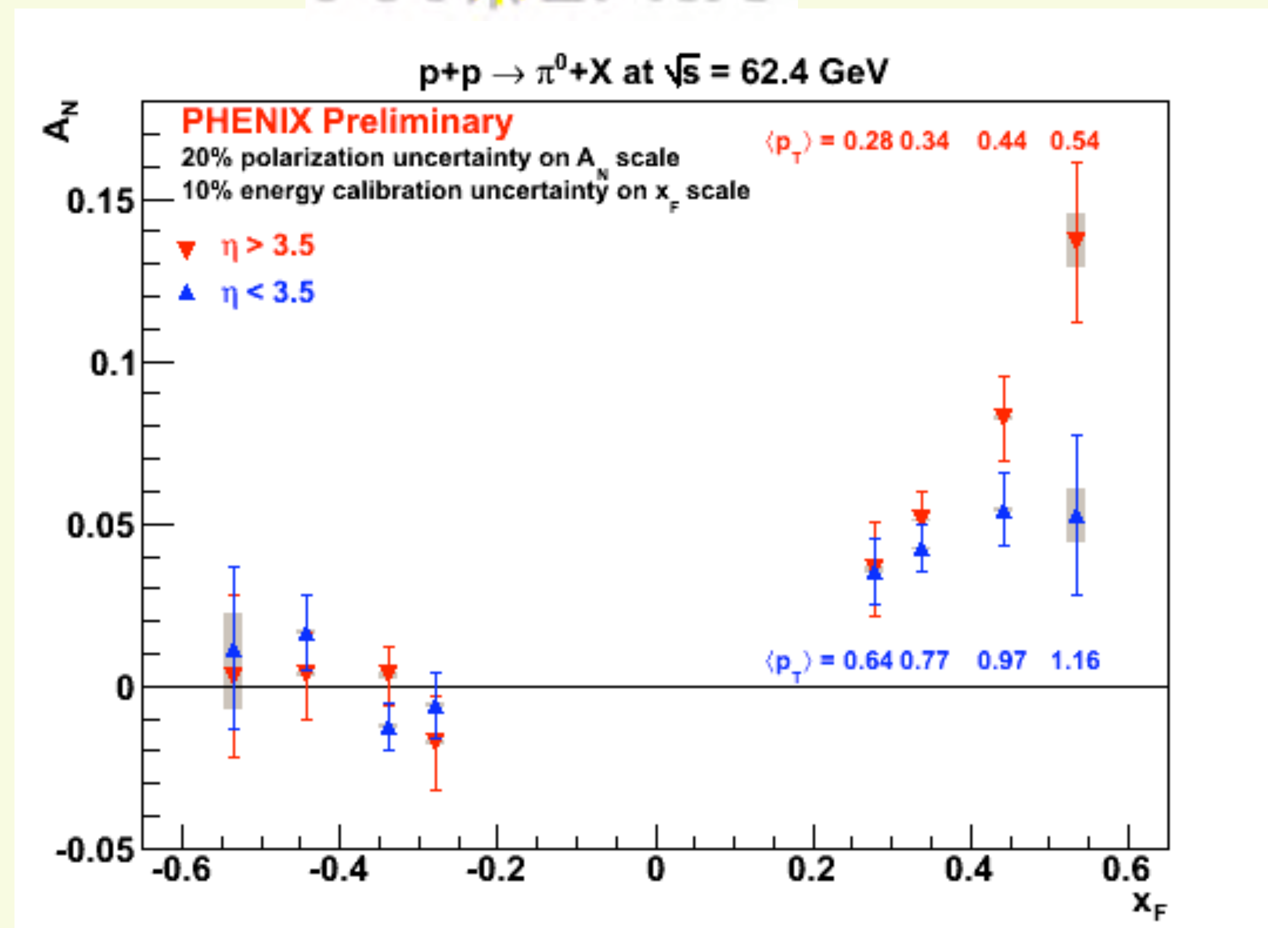
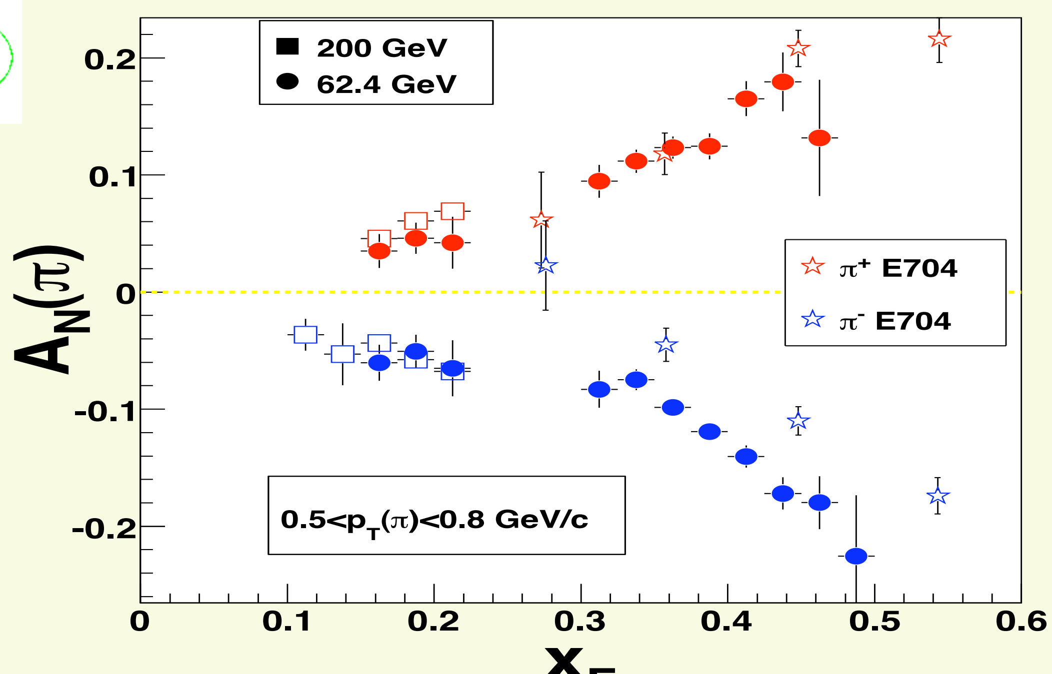
PRL101, 042001 (2008)

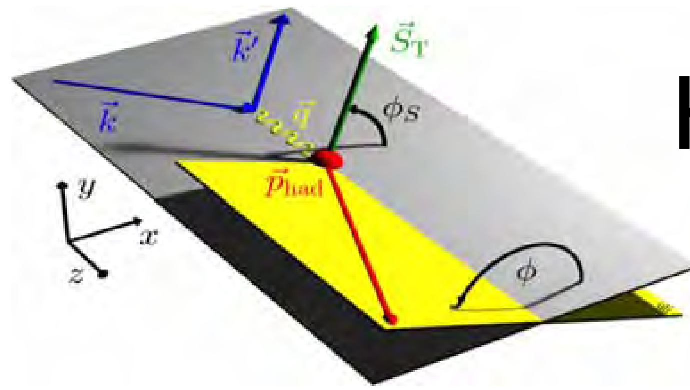
PHENIX



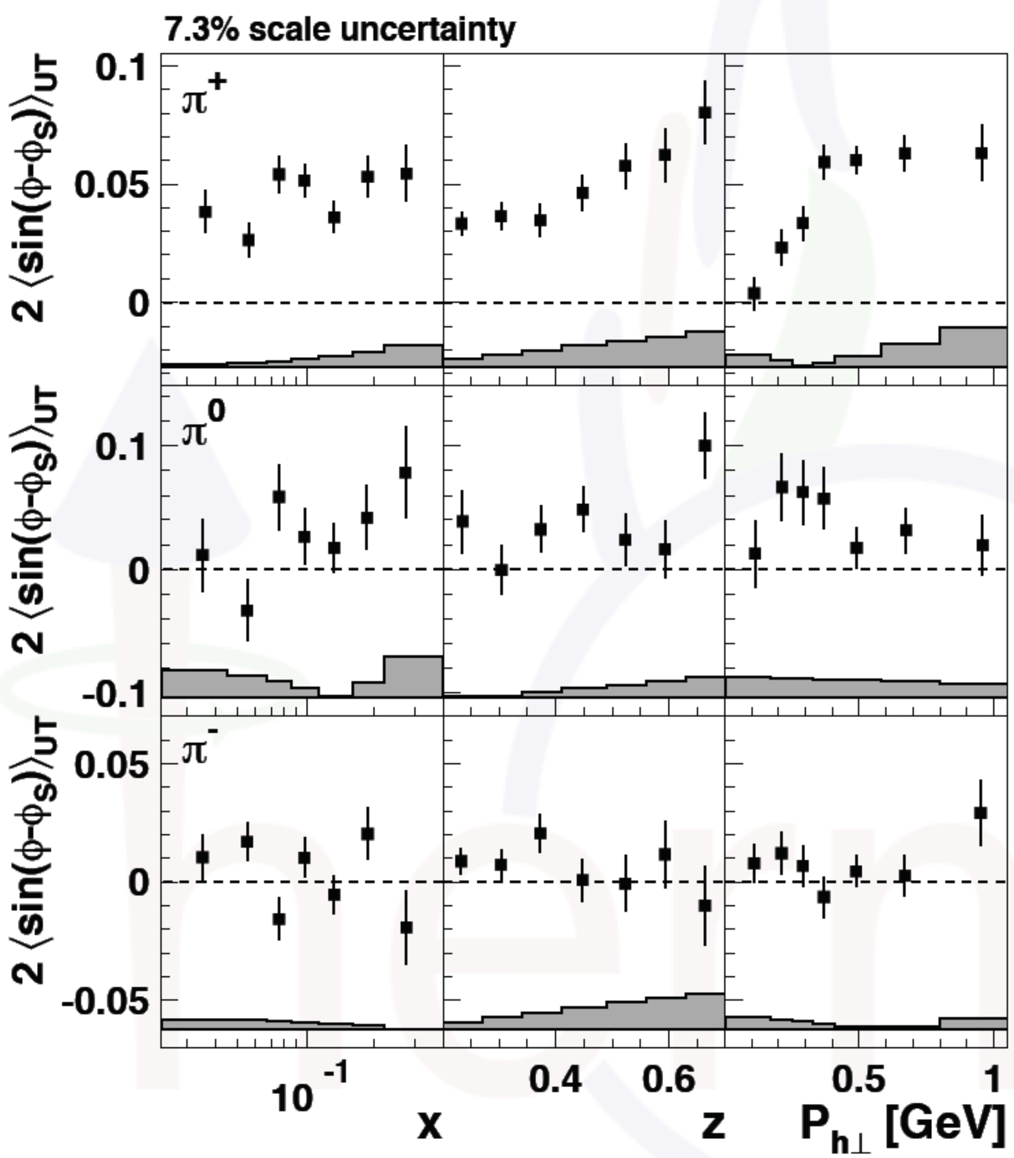
BRAHMS

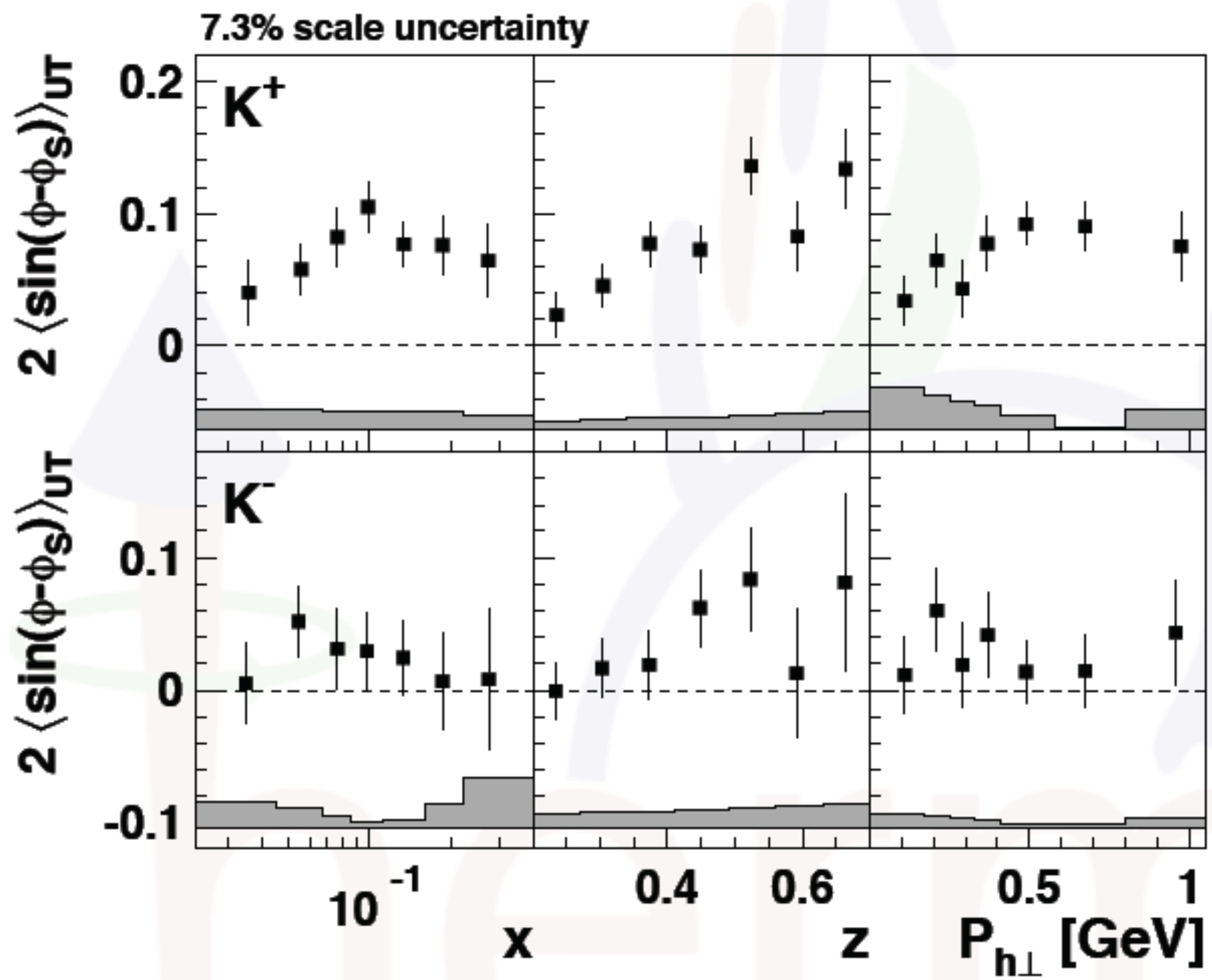


$$\ell p \to \ell' \, \pi X$$

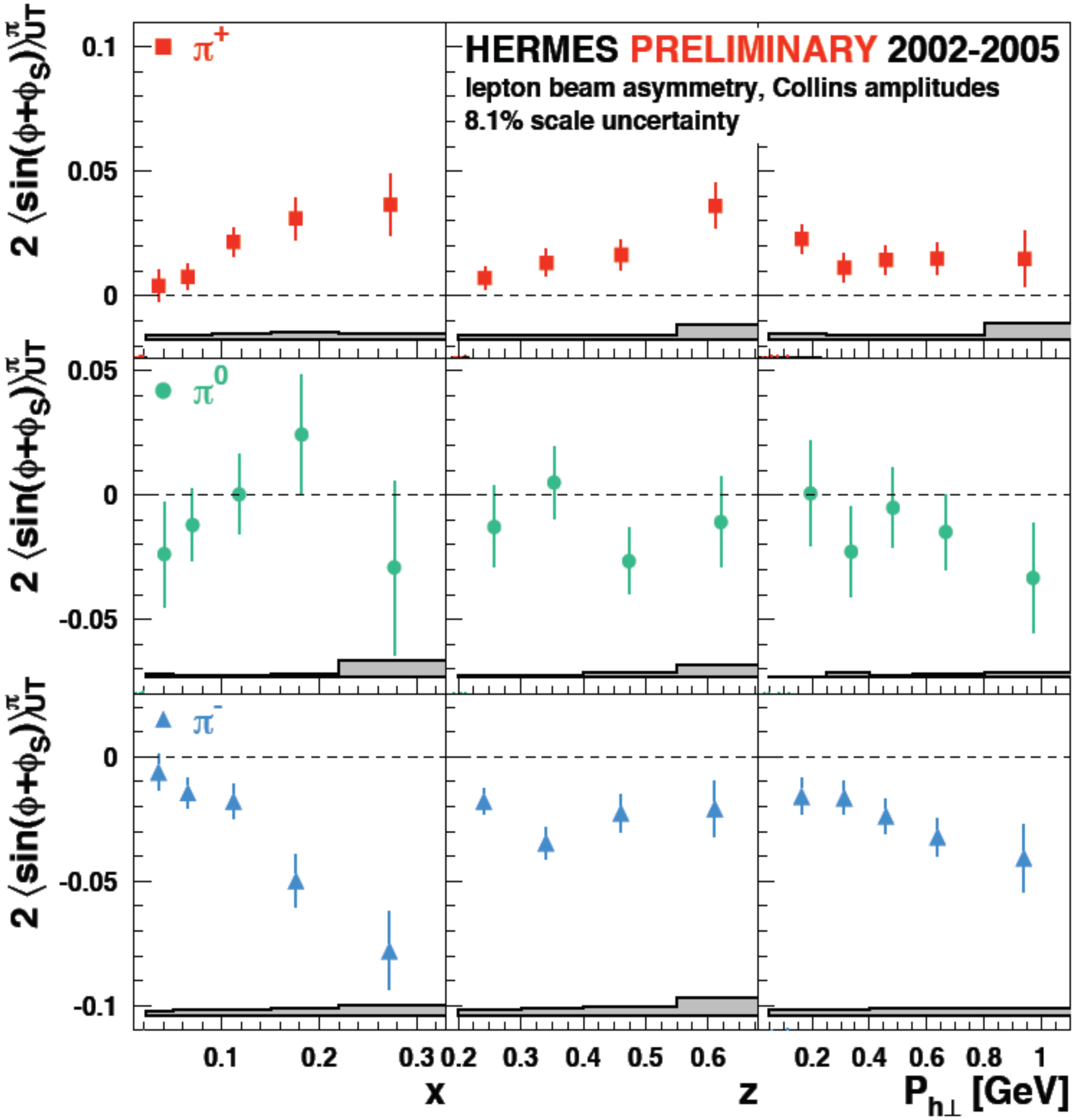


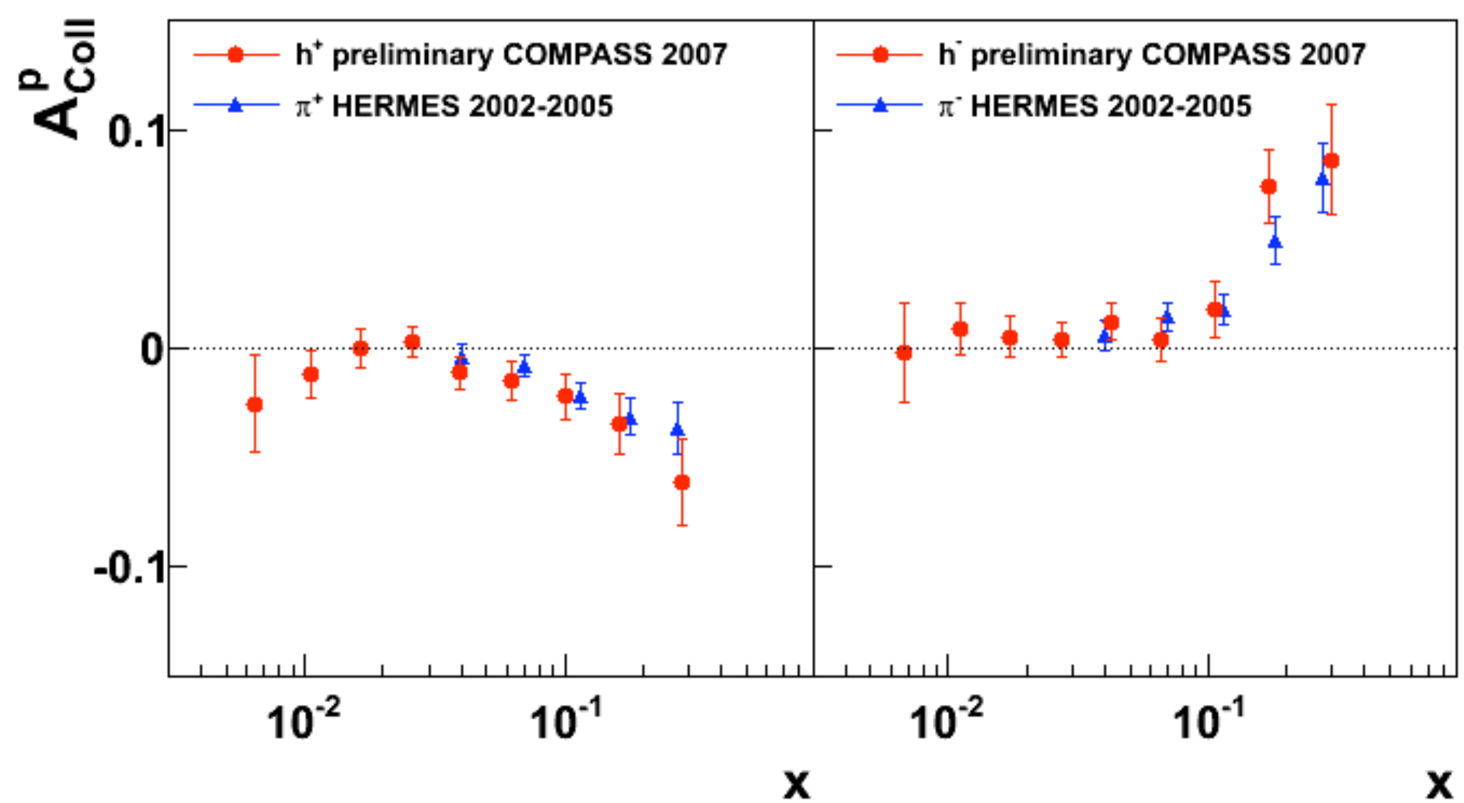
Hermes PRL 2009





Collins Asymmetry
Compass-proton data 2007 comparison w/ HERMES-Collins
D. Hasch INT-12 GeV

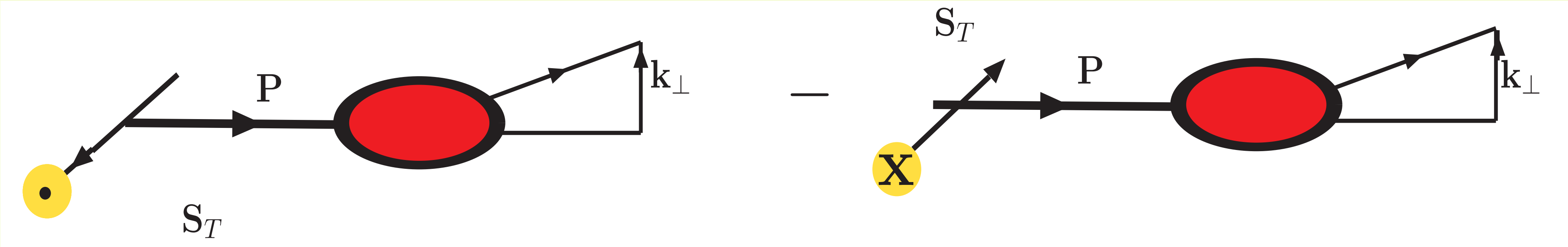




TSSAs thru "T-odd" non-pertb. **spin-orbit** *correlations....*

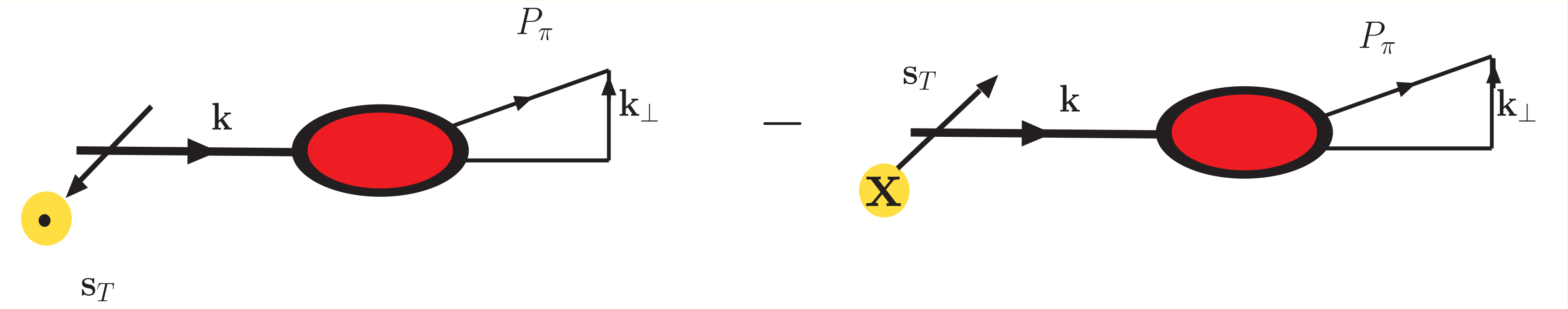
Sensitivity to $p_T \sim \mathbf{k}_T << \sqrt{Q^2}$

- **Sivers PRD: 1990** TSSA is associated w/ correlation *transverse* spin and momenta in initial state hadron



$$\Delta\sigma^{pp^\uparrow\to\pi X} \sim D \otimes f \otimes \Delta f^\perp \otimes \hat{\sigma}_{Born} \Longrightarrow \boldsymbol{\Delta f^\perp(x, k_\perp) = i S_T \cdot (P \times k_\perp)}\, f_{1T}^\perp(x, \boldsymbol{k}_\perp)$$

- **Collins NPB: 1993** TSSA is associated with *transverse* spin of fragmenting quark and transverse momentum of final state hadron



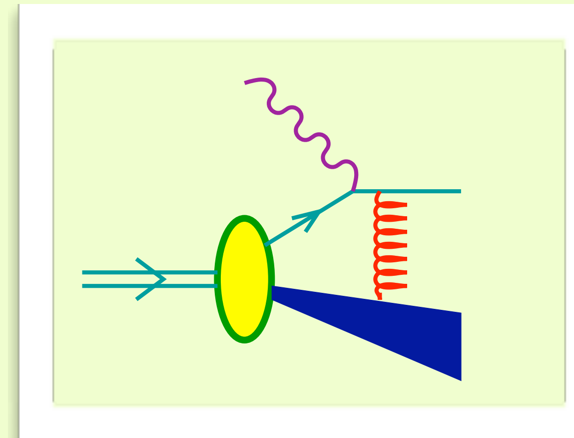
$$\Delta\sigma^{ep^\uparrow\to e\pi X} \sim \Delta D^\perp \otimes f \otimes \hat{\sigma}_{Born} \Longrightarrow \boldsymbol{\Delta D^\perp(x, p_\perp) = i s_T \cdot (P \times p_\perp)}\, H_1^\perp(x, \boldsymbol{p}_\perp)$$

Reaction Mechanism-FSI phases in TSSAs at unsupressed

- Brodsky, Hwang, Schmidt PLB: 2002

  SIDIS w/ transverse polarized nucleon target

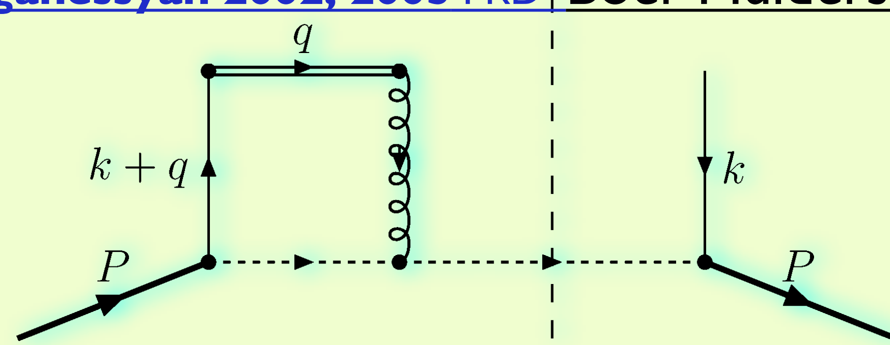
  $$e\ p^{\uparrow} \to e \pi X$$



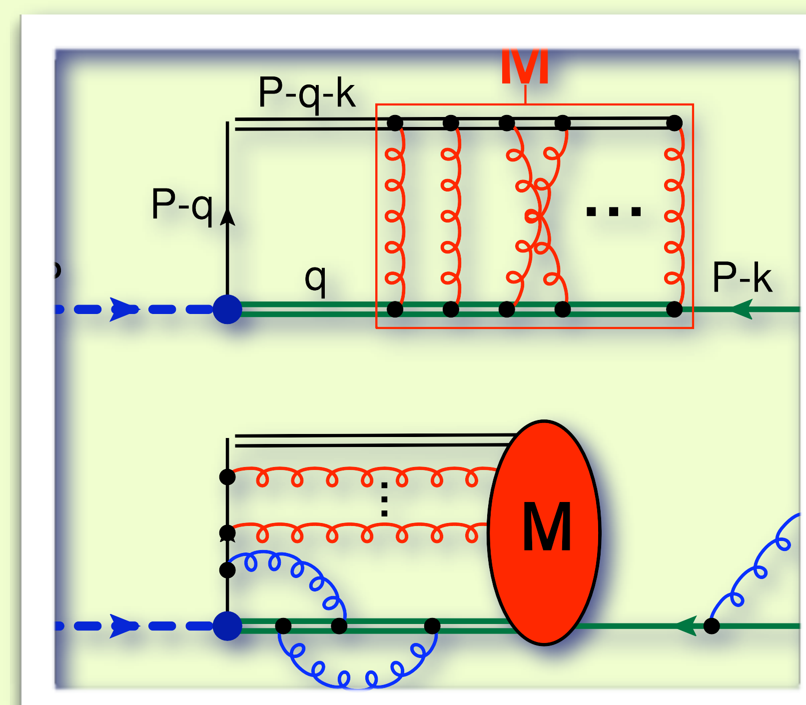
- ***Collins PLB 2002- Gauge link Sivers function doesn't vanish***

- Ji, Yuan PLB: 2002 -**Sivers fnct. FSI emerge from Color Gauge-links**

- **LG, Goldstein, Oganessyan 2002, 2003** PRD Boer-Mulders Fnct, and Sivers -spectator model



- **LG, M. Schlegel, PLB 2010** Boer-Mulders Fnct, and Sivers beyond summing the FSIs through the gauge link



Factorization & Sensitivity to $P_T \sim k_\perp$ $\longrightarrow$ TMDs

John Collins Nuclear Physics B396 (1993) 161–182

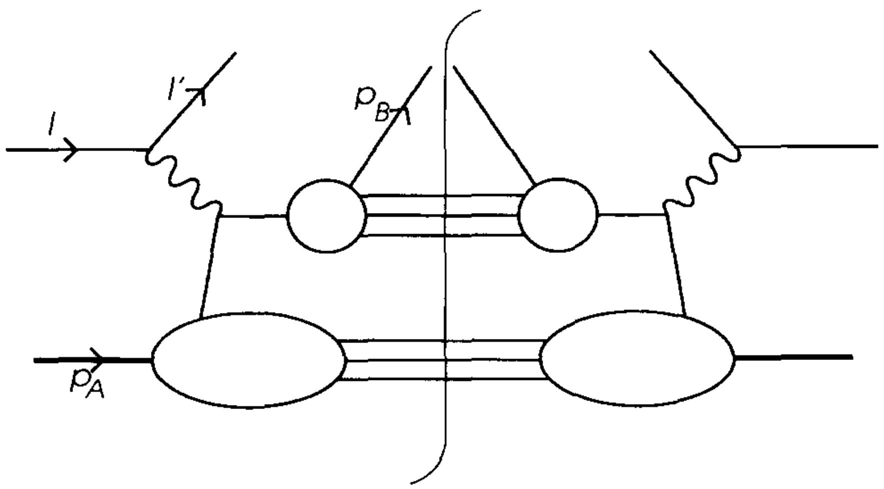


Fig. 2. Parton model for semi-inclusive deeply inelastic scattering.

3.4. FACTORIZATION WITH INTRINSIC TRANSVERSE MOMENTUM AND POLARIZATION

Ralston Soper NPB 1979, Collins NPB 1993

$$E'E_B \frac{\mathrm{d}\sigma}{\mathrm{d}^3l'\,\mathrm{d}^3p_B} = \sum_a \int \mathrm{d}\xi \int \frac{\mathrm{d}\zeta}{\zeta} \int \mathrm{d}^2k_{a\perp} \int \mathrm{d}^2k_{b\perp}\ \hat{f}_{a/A}(\xi, k_{a\perp})$$

$\longleftarrow$ Collins Soper NPB 1981, & Sterman NPB 1985

$$\times E'E_{k_b} \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}^3l'\,\mathrm{d}^3k_b} \hat{D}_{B/a}(\zeta, k_{b\perp}) + Y(x_{\mathrm{Bj}}, Q, z, q_\perp/Q)$$

The function $\hat{f}_{a/A}$ defined earlier gives the intrinsic transverse-momentum dependence of partons in the initial-state hadron. Similarly, $\hat{D}_{B/a}$ gives the distribution of hadrons in a parton, with $k_{b\perp}$ being the transverse momentum of the parton relative to the hadron.

Factorization parton model when P_T of the hadron small

$$W^{\mu\nu}(q,P,S,P_h) \approx \sum_a e^2 \int \frac{d^2\mathbf{p}_T dp^- dp^+}{(2\pi)^4} \int \frac{d^2\mathbf{k}_T dk^- dk^+}{(2\pi)^4} \delta(p^+ - x_B P^+)\delta(k^- - \frac{P_h^-}{z})\delta^2(\mathbf{p}_T + \mathbf{q}_T - \mathbf{k}_T)$$

$$\times \mathrm{Tr}\left[\Phi(p,P,S)\gamma^\mu \Delta(k,P_h)\gamma^\nu\right]$$

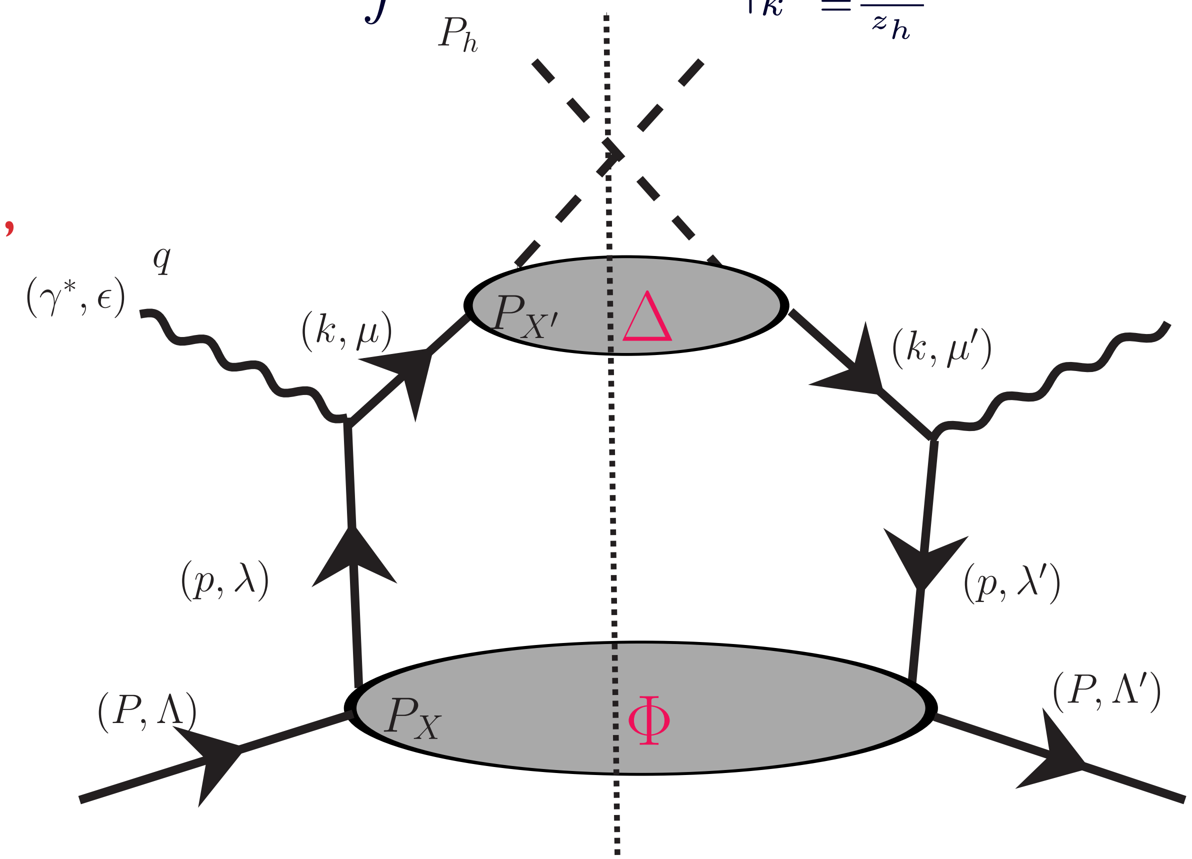
$$W^{\mu\nu}(q,P,S,P_h) = \int \frac{d^2\mathbf{p}_T}{(2\pi)^4} \int \frac{d^2\mathbf{k}_T}{(2\pi)^4} \delta^2(\mathbf{p}_T - \frac{\mathbf{P}_{h\perp}}{z} - \mathbf{k}_T)\mathrm{Tr}\left[\left(\int dp^-\Phi\right)\gamma^\mu \left(\int dk^+\Delta\right)\gamma^\nu\right]$$

Integrate out small longitudinal momenta components

Small transverse momentum

$$\Phi(x,\mathbf{p}_T,S) \equiv \int dp^- \Phi(p,P,S)\Big|_{p^+=x_B P^+}, \qquad \Delta(z,\mathbf{k}_T) \equiv \int dk^+ \Delta(k,P_h)\Big|_{k^-=\frac{P^-}{z_h}}$$

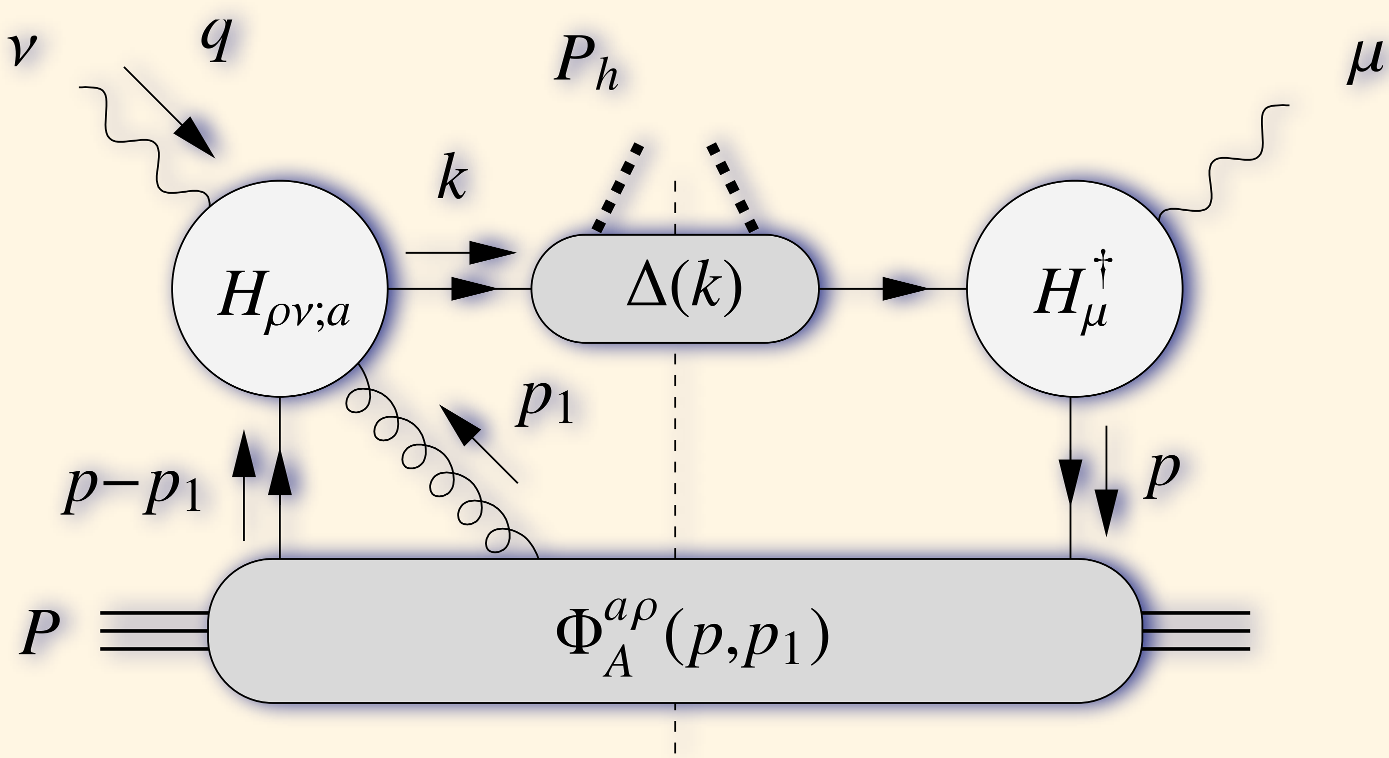
Integration support for integrals is where transverse momentum is small-"cov parton model"
e.g. Landshoff Polkinghorne NPB28, 1971



What about FSIs and TSSAs?
Extend Parton Model result-Gauge Links

- What are the "leading order" gluons that implement color gauge invariance?
- How is the correlator modified?

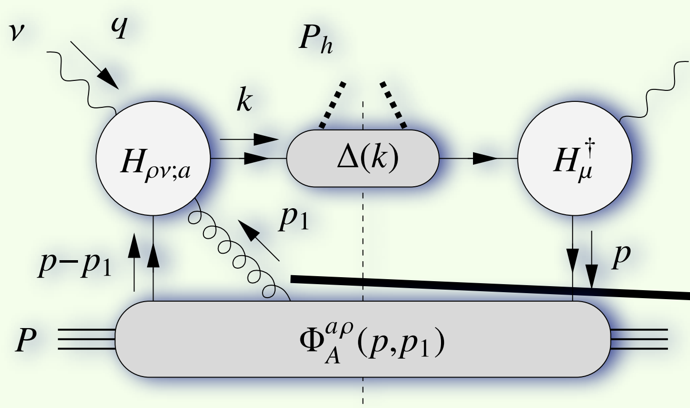
$$H_{\rho,\nu} = \gamma^{\nu}$$

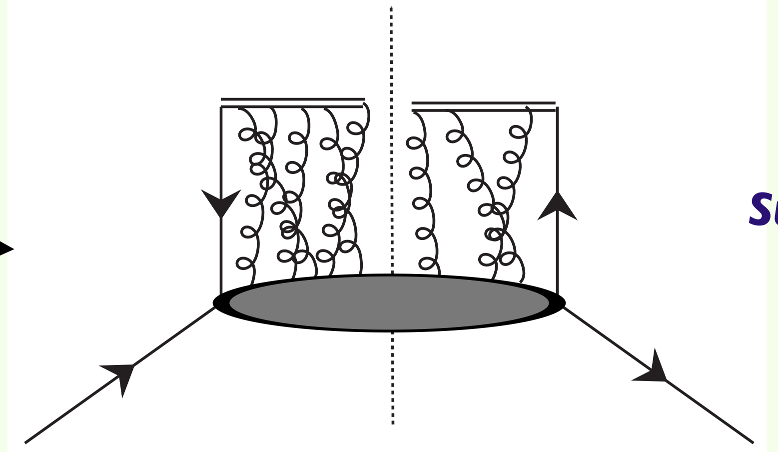


“T-Odd” Effects From Color Gauge Inv. Via Gauge links

Gauge link determined re-summing gluon interactions btwn soft and hard
Efremov,Radyushkin *Theor. Math. Phys. 1981*
Belitsky, Ji, Yuan NPB 2003,
Boer, Bomhof, Mulders Pijlman, et al. 2003 - 2008- NPB, PLB, PRD

$$\Phi^{[\mathcal{U}[C]]}(x,p_T) = \int \frac{d\xi^- d^2\xi_T}{2(2\pi)^3} e^{ip\cdot\xi} \langle P|\overline{\psi}(0)\mathcal{U}^{[C]}_{[0,\xi]}\psi(\xi^-,\xi_T)|P\rangle|_{\xi^+=0}$$

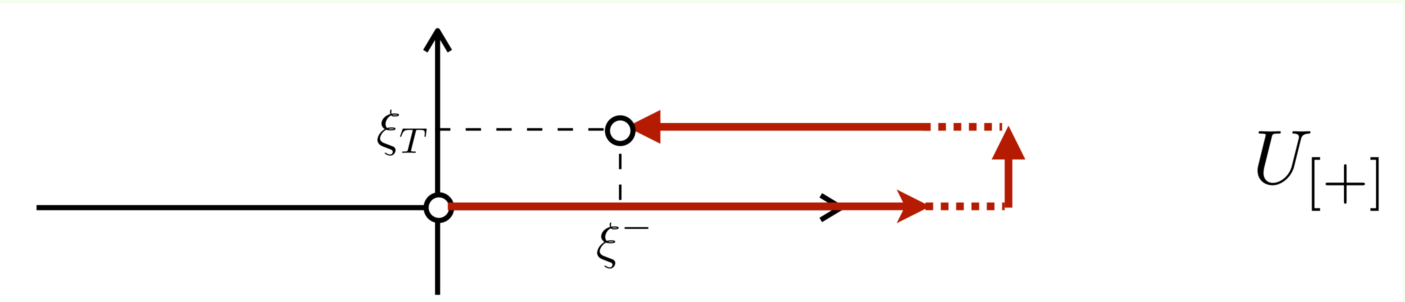




Summing gauge link with color
LG, M. Schlegel PLB 2010

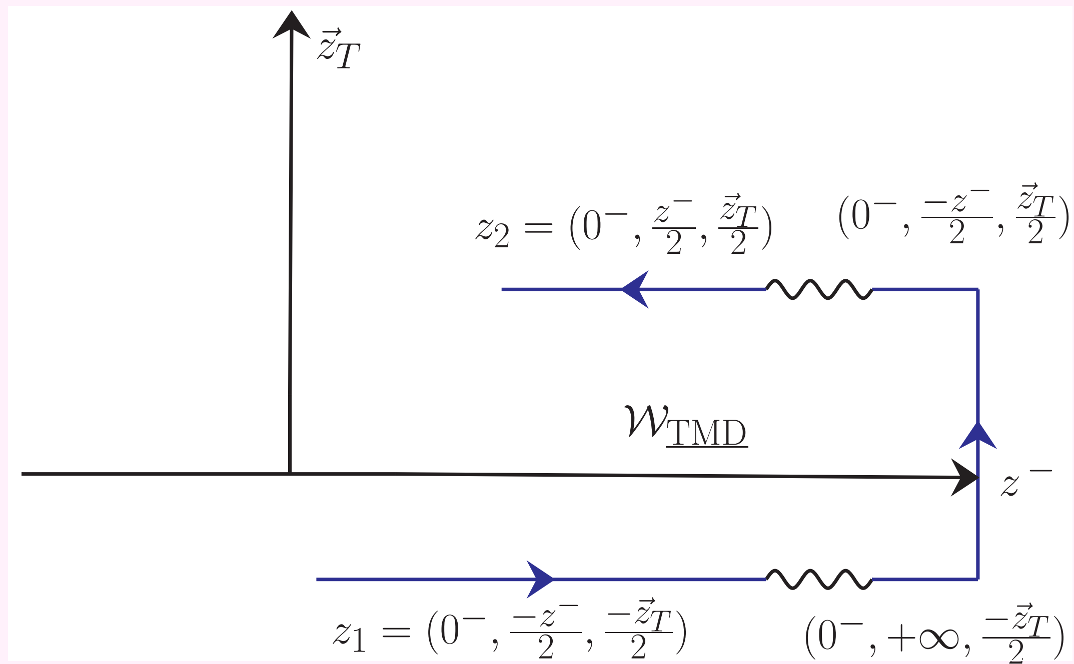
- **The path $[C]$ is fixed by hard subprocess within hadronic process.**

$$\int d^4p d^4k \delta^4(p+q-k)\mathrm{Tr}\left[\Phi^{[U^{C}_{[\infty;\xi]}}(p)H^\dagger_\mu(p,k)\Delta(k)H_\nu(p,k)\right]$$

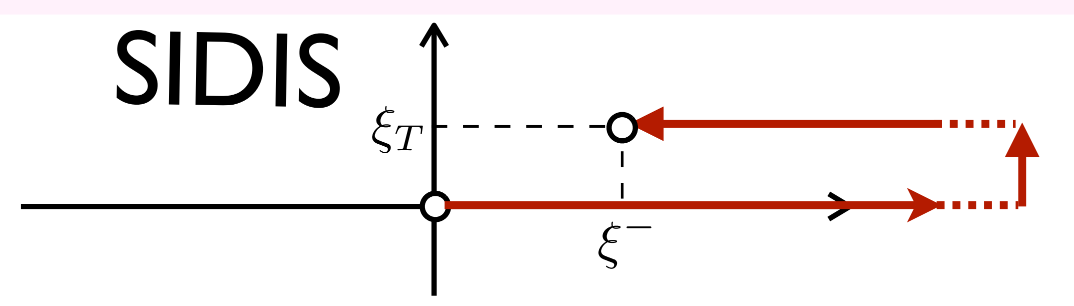


Wilson Line = Gauge links ... Path ordered Eikonal

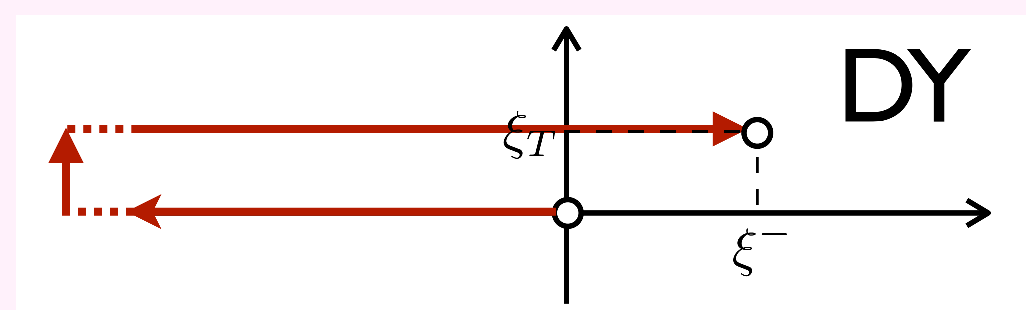
$$U^s_{[z_1,z_2]} = \mathcal{W}[z_1; z_2] = [z_1; z_2] = \mathcal{P}\mathrm{e}^{-ig\int_{z_1}^{z_2} ds\cdot A(s)}$$



Process Dependence, Collins plb 02, Brodsky et al. NPB 02, Boer Mulders Pijlman Bomhoff 03, 04 ...

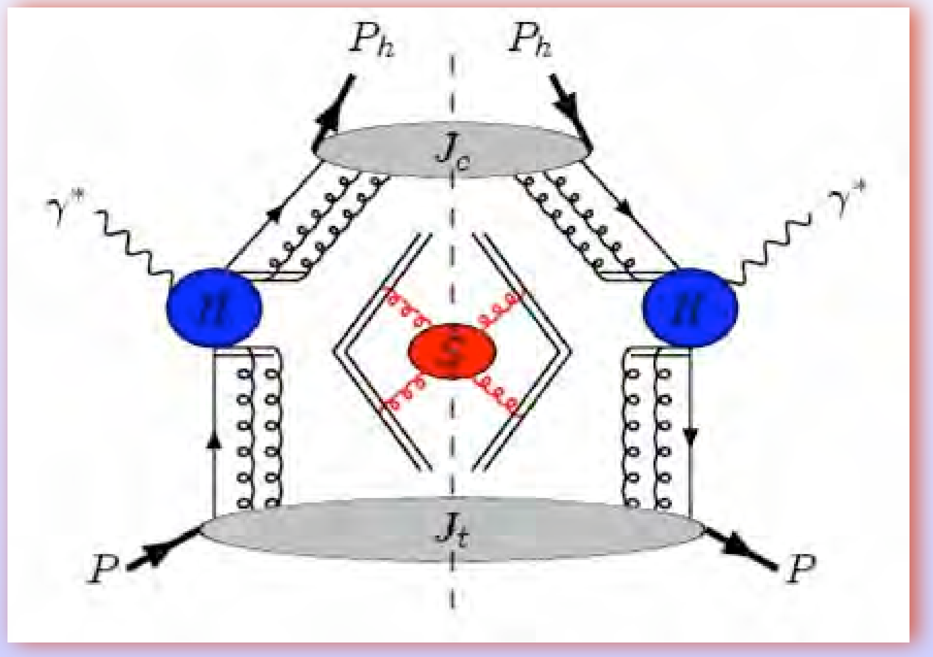


P&T



$$\Phi^{[+]*}(x, p_T) = i\gamma^1\gamma^3\Phi^{[-]}(x, p_T)i\gamma^1\gamma^3$$

Ji, Ma, Yuan: PLB, PRD 2004, 2005 Extend factorization of CS-NPB: 81



Also see Bacchetta Boer Diehl Mulders JHEP 08

$$F_{UU,T}(x,z,P_{h\perp}^2,Q^2) = \mathcal{C}\big[f_1 D_1\big]$$

$$= \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, d^2\boldsymbol{l}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T - \boldsymbol{P}_{h\perp}/z\big)$$

$$x \sum_a e_a^2\, f_1^a(x,p_T^2,\mu^2)\, D_1^a(z,k_T^2,\mu^2)\, U(l_T^2,\mu^2) H(Q^2,\mu^2)$$

TMD PDF TMD FF Soft factor Hard part

Collins, Soper, NPB 193 (81)
Ji, Ma, Yuan, PRD 71 (05)

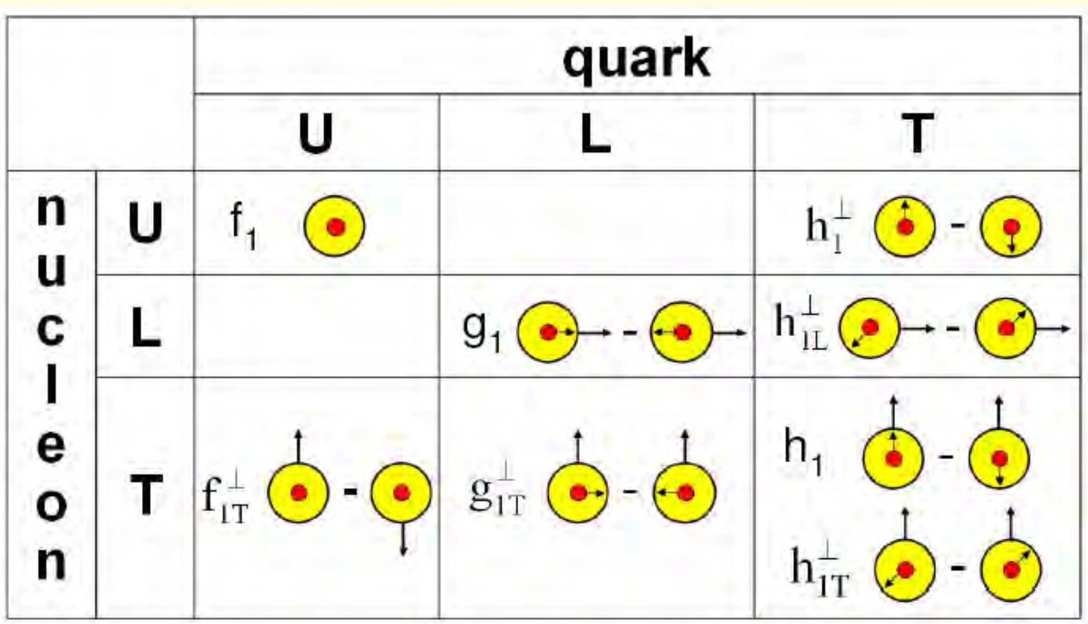
Leading Twist TMDs from Correlator is Matrix in Dirac space

$$\Phi^{[\gamma^+]}(x, \boldsymbol{p}_T) \equiv f_1(x, \boldsymbol{p}_T^2) + \frac{\epsilon_T^{ij} p_{Ti} S_{Tj}}{M} f_{1T}^\perp(x, \boldsymbol{p}_T^2)$$

$$\Phi^{[\gamma^+\gamma_5]}(x, \boldsymbol{p}_T) \equiv \lambda\, g_{1L}(x, \boldsymbol{p}_T^2) + \frac{\boldsymbol{p}_T \cdot \boldsymbol{S}_T}{M} g_{1T}(x, \boldsymbol{p}_T^2)$$

$$\Phi^{[i\sigma^{i+}\gamma_5]}(x, \boldsymbol{p}_T) \equiv S_T^i\, h_{1T}(x, \boldsymbol{p}_T^2) + \frac{p_T^i}{M}\left(\lambda\, h_{1L}^\perp(x, \boldsymbol{p}_T^2) + \frac{\boldsymbol{p}_T \cdot \boldsymbol{S}_T}{M} h_{1T}^\perp(x, \boldsymbol{p}_T^2)\right)$$

$$+ \frac{\epsilon_T^{ij} p_T^j}{M} h_1^\perp(x, \boldsymbol{p}_T^2)$$



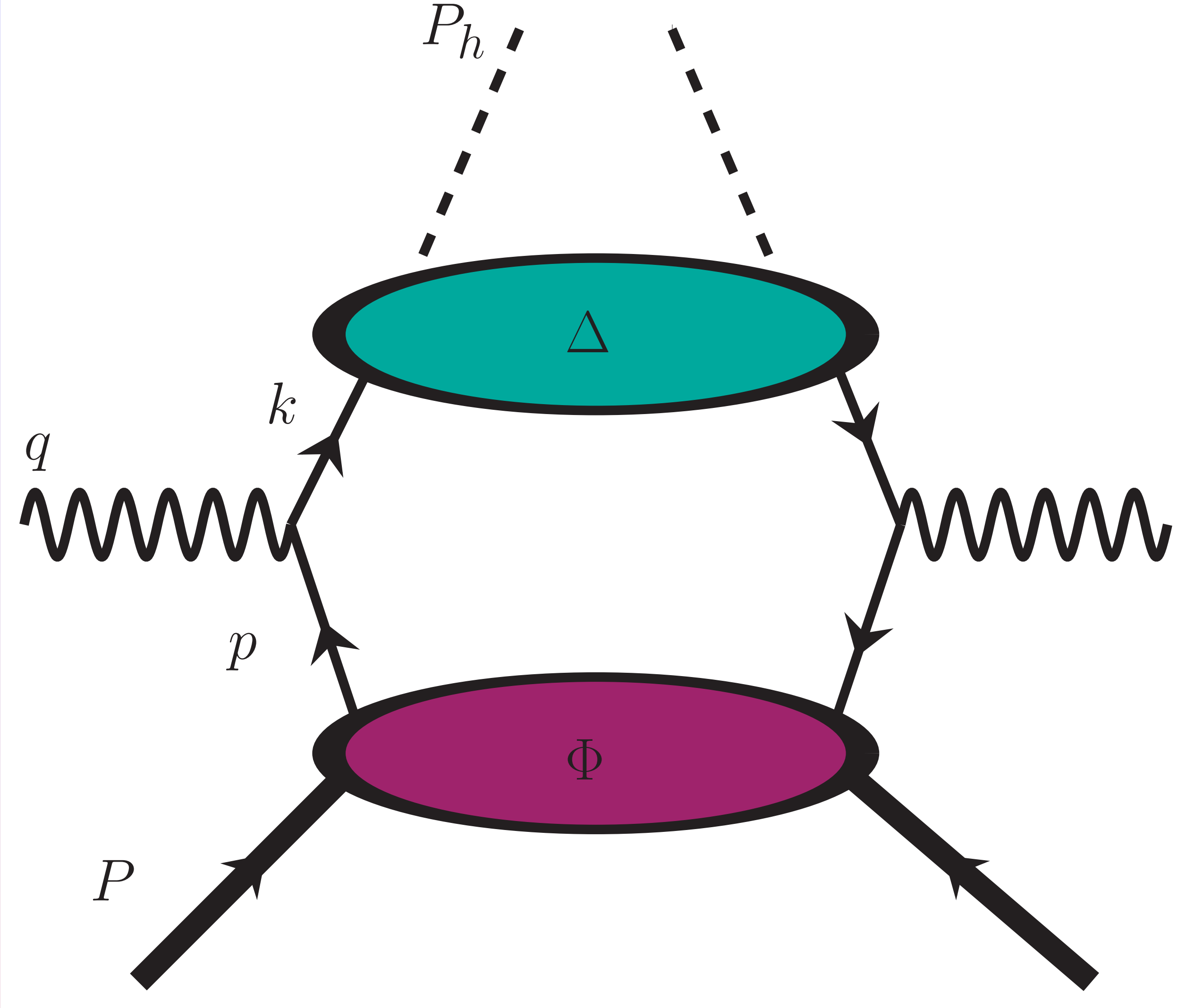
“Avakian Mulders-tableau”

TSSAs in SIDIS

$$d^6\sigma = \hat{\sigma}_{\text{hard}}\,\mathcal{C}[w f D]$$

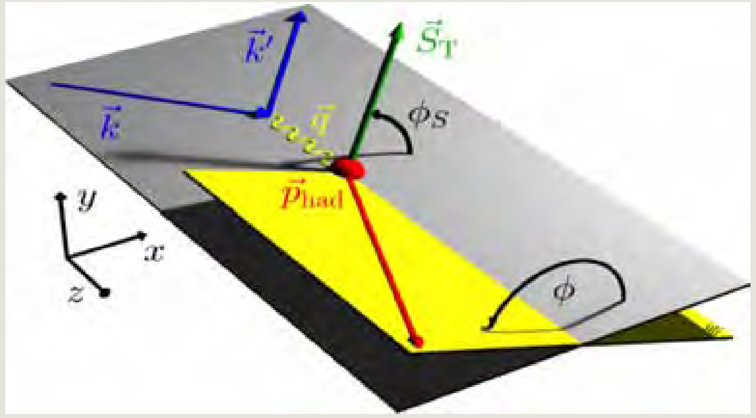
Structure functions that are extracted

$$\mathcal{F}_{AB} = \mathcal{C}[w\, f\, D]$$



$$\mathcal{C}[wfD] = \sum_a x e_a^2 \int d^2\mathbf{p}_T d^2\mathbf{k}_T \delta^{(2)}\left(\mathbf{p}_T - \mathbf{k}_T - \frac{P_{h\perp}}{z}\right) f^a(x, p_T^2) D^a(z, k_T^2)$$

Transverse Spin Observables and TMD Correlators in SIDIS



$$\Phi(x,\boldsymbol{p}_T)=\frac{1}{2}\Big\{f_1(x,\boldsymbol{p}_T)\not{P}+ih_1^{\perp}(x,\boldsymbol{p}_T)\frac{[\not{p}_T,\not{P}]}{2M}-f_{1T}^{\perp}(x,\boldsymbol{p}_T)\frac{\epsilon_T^{ij}p_{Ti}S_{Tj}}{M}\not{P}\cdots\Big\}$$

$$\Delta(z,\boldsymbol{k}_T)=\frac{1}{4}\Big\{zD_1(z,\boldsymbol{k}_T)\not{P}_h+izH_1^{\perp}(z,\boldsymbol{k}_T)\frac{[k_T,\not{P}_h]}{2M_h}-zD_{1T}^{\perp}(z,\boldsymbol{k}_T)\frac{\epsilon_T^{ij}k_{Ti}S_{Tj}}{M_h}\not{P}_h+\cdots\Big\}$$

SIDIS cross section

$$\begin{aligned}
d\sigma^{\ell N\to\ell\pi X}_{\{\lambda,\Lambda\}} \quad \propto \quad & f_1\otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes D_1+\frac{k_\perp}{Q}f_1\otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes D_1\cdot\cos\phi \\
+ \quad & \left[\frac{k_\perp^2}{Q^2}f_1\otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes D_1+h_1^{\perp}\otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes H_1^{\perp}\right]\cdot\cos 2\phi \\
+ \quad & |S_T|\cdot h_1\otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes H_1^{\perp}\cdot\sin(\phi+\phi_S) \quad \text{Collins} \\
+ \quad & |S_T|\cdot f_{1T}^{\perp}\otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes D_1\cdot\sin(\phi-\phi_S) \quad \text{Sivers} \\
+ \quad & |S_L|\cdot h_{1L}^{\perp}\otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes H_1^{\perp}\cdot\sin(2\phi) \quad \text{Kotzinian-MuldersPLB}
\end{aligned}$$

Boer-Mulders

transversity

Spec. model workbench for ISI/FSI TSSAs &TMDs $f_{1T}^{\perp}, h_1^{\perp}, D_{1T}^{\perp}, H_1^{\perp}$ *& gluonic pole MEs*

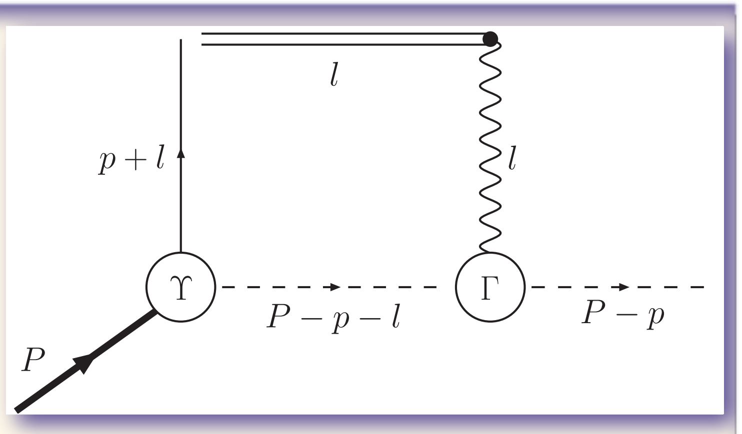
- $\not\exists$ *calculation Quark-Quark Correlator in Full QCD*

$$\Phi^{[\mathcal{U}[\mathcal{C}]]}(x, p_T) = \int \frac{d\xi^- d^2\xi_T}{2(2\pi)^3} e^{ip\cdot\xi} \langle P|\overline{\psi}(0)\mathcal{U}^{[C]}_{[0,\xi]}\psi(\xi^-, \xi_T)|P\rangle|_{\xi^+=0}$$

- Use Spectator Framework Develop a QFT to explore and estimates these effects with gauge links
  - ⋆ BHS FSI/ISI Sivers fnct, -PLB 2002, NPB 2002
  - ⋆ Ji, Yuan PLB 2002 - Sivers Function
  - ⋆ Metz PLB 2002 - Collins Function
  - ⋆ L.G. Goldstein, 2002 ICHEP- Boer Mulders Function
  - ⋆ L.G. Goldstein, Oganessyan TSSA & AAS PRD 2003-SIDIS
  - ⋆ Boer Brodsky Hwang PRD 2003-Drell Yan Boer Mulders
  - ⋆ Bacchetta Jang Schafer 2004- PLB, Flavor-Sivers, Boer Mulders
  - ⋆ Lu Ma Schmidt PLB, PRD, 2004/2005 Pion Boer Mulders
  - ⋆ L.G. Goldstein DY and higher twist, PLB 2007
  - ⋆ LG, Goldstein, Schlegel PRD 2008-Flavor dep. Boer Mulders $\cos 2\phi$ SIDIS
  - ⋆ Conti, Bacchetta, Radici, Ellis, Hwang, Kotzinian 2008 hep-ph . . . !
- Spectator Model "Field Theoretic" used study Universality of T-odd Fragmentation Δ_{ij}
  - ⋆ Metz PLB 2002, Collins Metz PRL 2004
  - ⋆ Bacchetta, Metz, Yang, PBL 2003, Amrath, Bacchetta, Metz 2005,
  - ⋆ Bacchetta, L.G. Goldstein, Mukherjee, PLB 2008
  - ⋆ Collins Qui, Collins PRD 2007,2008
  - ⋆ Yuan 2-loop Collins function PRL 2008
  - ⋆ L.G., Mulders, Mukherjee Gluonic Poles PRD 2008
  - ★ **Mulders & Rogers Fact. breaking PRD 2010**

Studies FSIs in 1-gluon exchange approx.

LG, G. Goldstein, M. Schlegel PRD 77 2008, Bacchetta Conti Radici PRD 78, 2008



Build the T-odd TMD PDF with Final State Interactions-- one gluon exchange approx of Gauge link

$$W_i(P,k,S) = -ie_q e_{dq}\int \frac{d^4 l}{(2\pi)^4}\frac{g_{ax}((p+l)^2)}{\sqrt{3}}\varepsilon^*_\sigma(P-p,\lambda)\mathcal{D}^{ax}_{\rho\eta}(P-p-l)$$
$$\times\frac{[g^{\sigma\rho}v\cdot(2P-2p-l)+(1+\kappa)(v^\sigma(P-p+l)^\rho+v^\rho(P-p-2l)^\sigma)]}{[l\cdot v+i0][l^2+i0][(l+p)^2-m_q^2+i0]}$$
$$\times\Big[(\not p+\not l+m_q)\gamma_5\Big(\gamma^\eta-R_g\frac{P^\eta}{M}\Big)u(P,S)\Big]_i,$$

$$\epsilon_T^{ij}k_T^i S_T^j f_{1T}^\perp(x,\vec{k}_T^2) = -\frac{M}{8(2\pi)^3(1-x)P^+}\left(\bar{W}\gamma^+W\Big|_{S_T} - \bar{W}\gamma^+W\Big|_{-S_T}\right)$$

Many model calculations studying dynamics of FSIs Brodsky, Hwang et al, Bacchetta & Radici, et al, Pasquini et al, Courtoy et al,

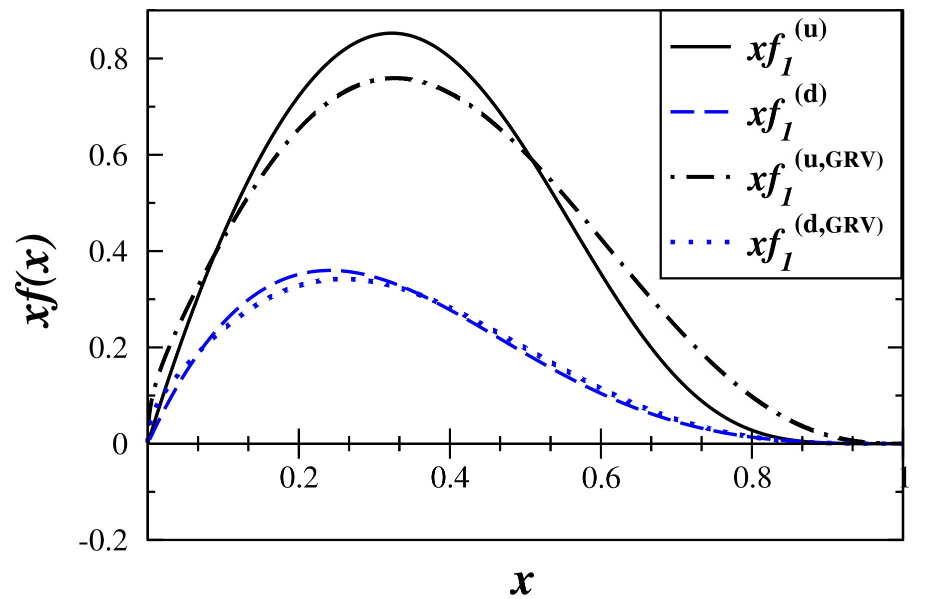
....

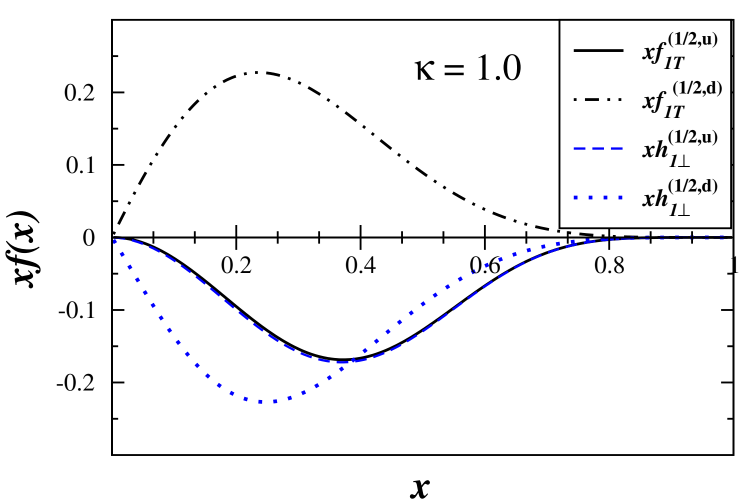
Flavor Dependence: Results & Phenomenology

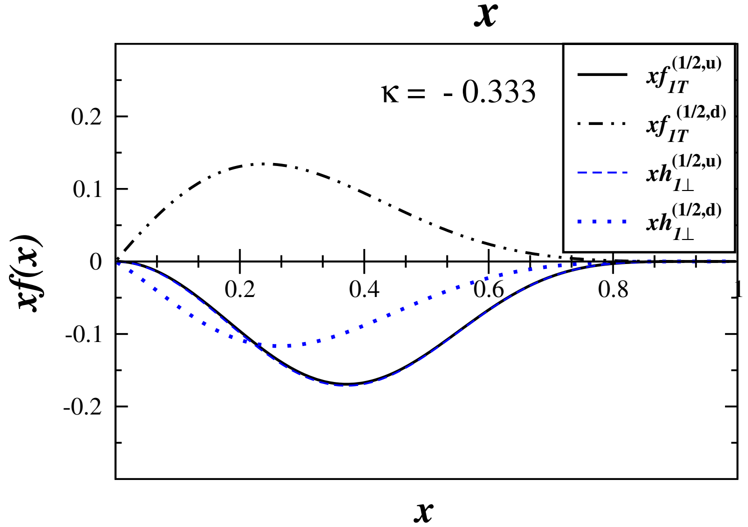
Flavor-dependent PDFs from diquark models: $u = \frac{3}{2}s + \frac{1}{2}a$, $d = a$,

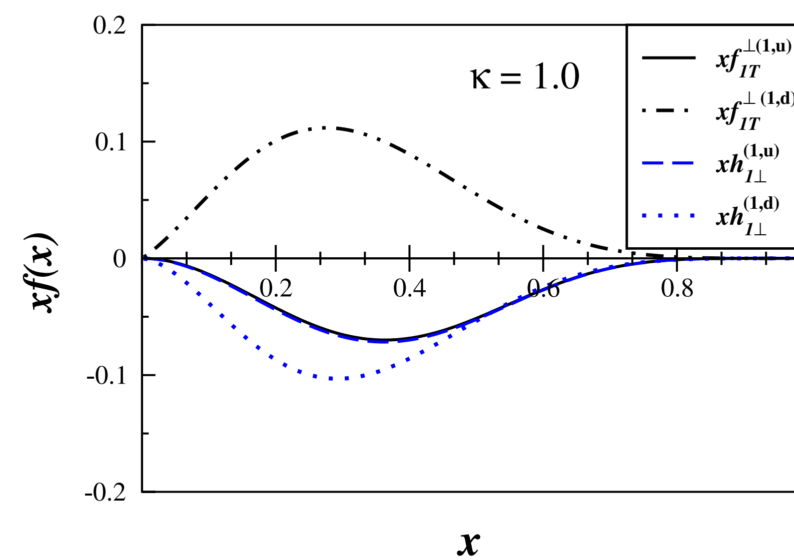
moments: $h_1^{\perp(1/2)}(x) = \int d^2\vec{p}_T \, \frac{|\vec{p}_T|}{M} h_1^\perp(x, \vec{p}_T^{\,2})$ L.G. Goldstein, Schlegel PRD 2008

- **Comparison to** $f_1^{(u,d)}$ **(Glück, Reya, Vogt)** $\rightarrow$ **parameters of the model,**









- **Boer Mulders up and down are negative is spectator model and** $f_{1T}^{(u)} \sim h_1^{\perp(u)}$

Sivers

Anselmino et al. PRD 05, EPJA 08

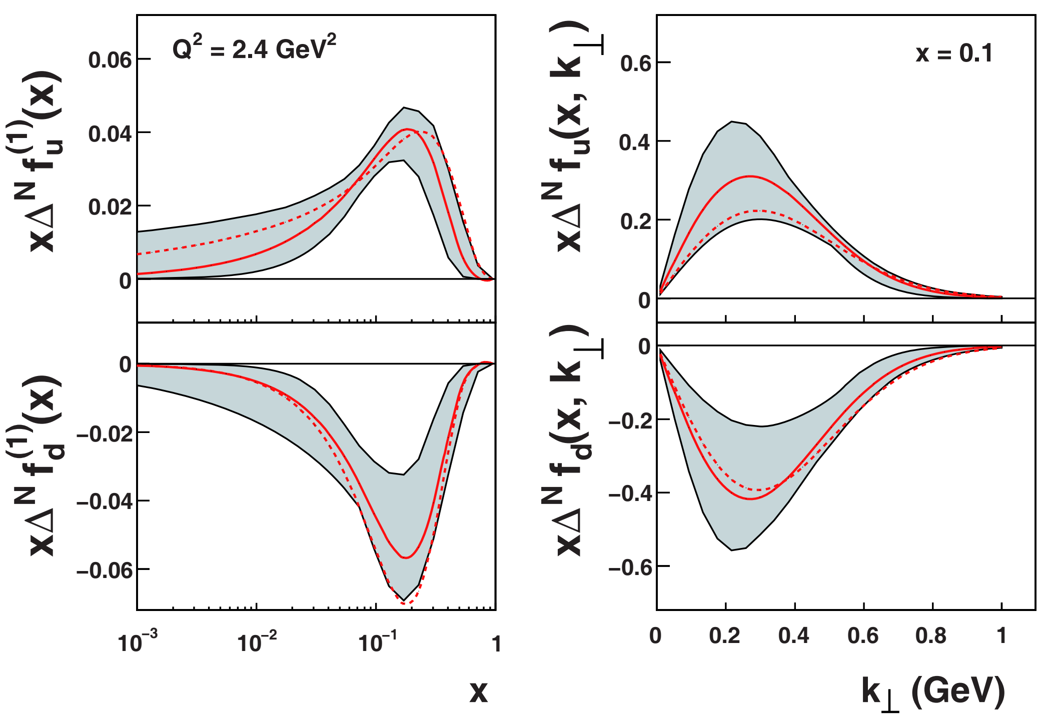


Fig. 7. The Sivers distribution functions for u and d flavours, at the scale $Q^2 = 2.4\,(\mathrm{GeV}/c)^2$, as determined by our present fit (solid lines), are compared with those of our previous fit [2] of SIDIS data (dashed lines), where π^0 and kaon productions were not considered and only valence quark contributions were taken into account. This plot clearly shows that the Sivers functions previously found are consistent, within the statistical uncertainty bands, with the Sivers functions presently obtained.

Gamberg, Goldstein,Schlegel PRD 77, 2008

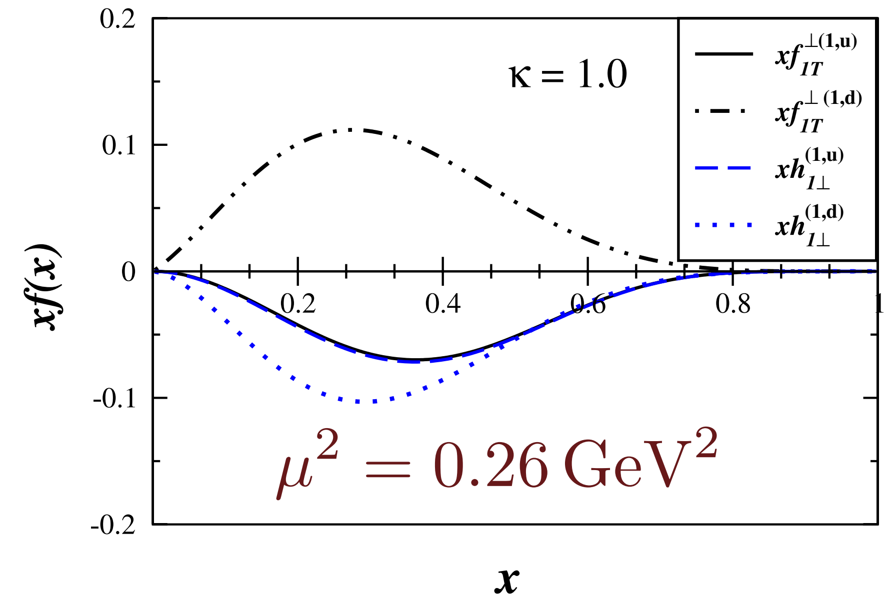
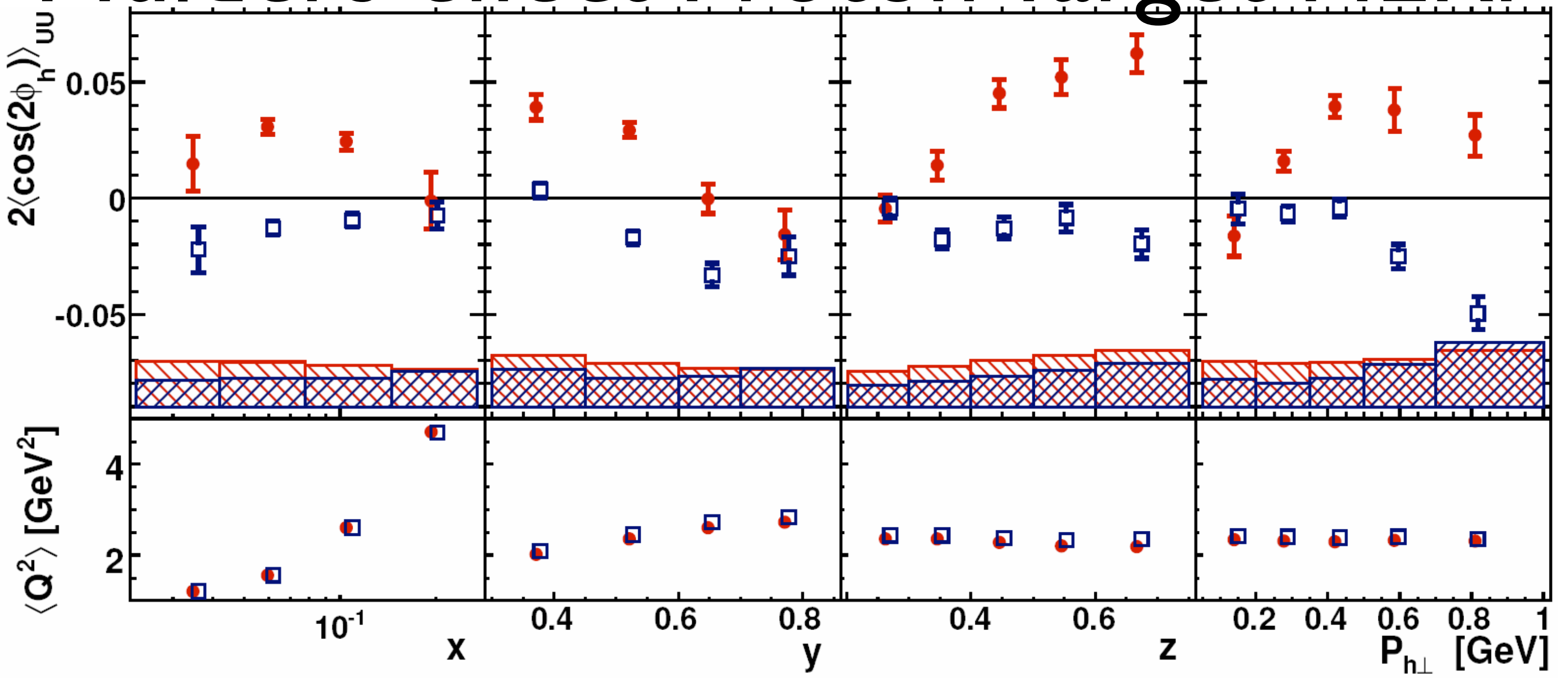
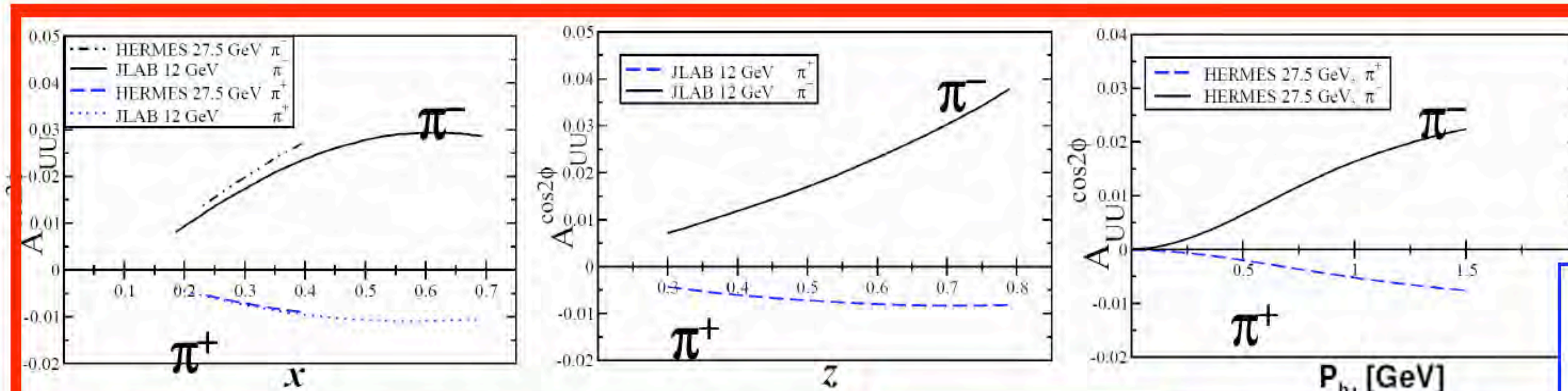


FIG. 5 (color online). The first moment of the Boer-Mulders and Sivers functions versus x for $\kappa = 1.0$.

Boer Mulders effect Proton Target HERMES

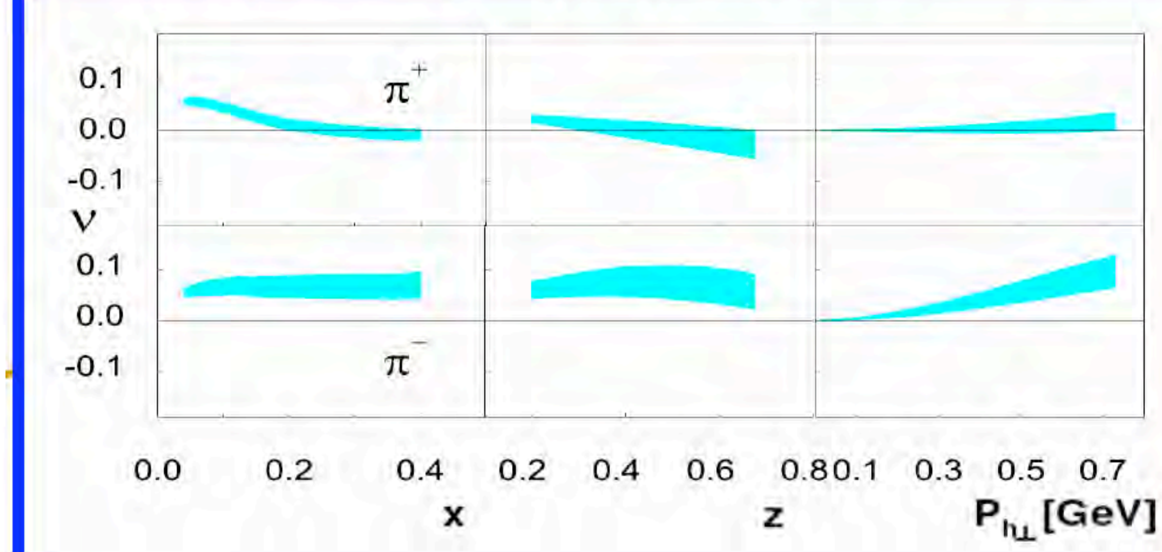
L Pappalardo-jlab exclusive wksp

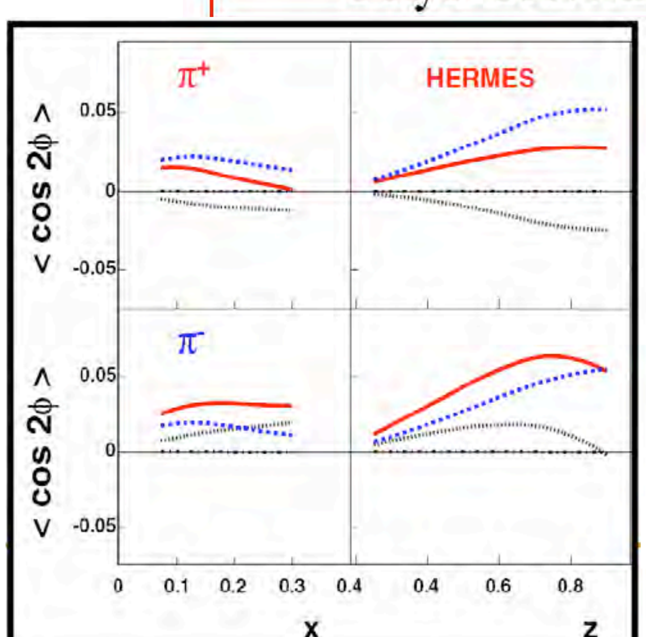




L. P. Gamberg and G. R. Goldstein, Phys. Rev. D77:094016, 2008

B. Zhang et al., Phys. Rev. D78:034035, 2008





V. Barone et al. Phys.Rev. D78:045022, 2008

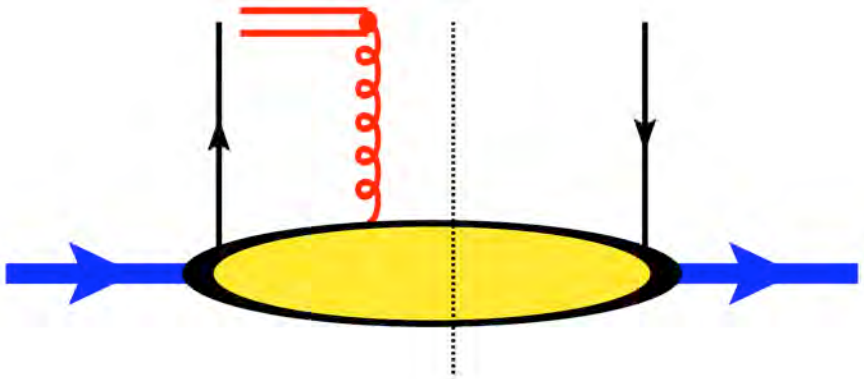
All contributions

Boer-Mulders

Cahn (twist 4)

Beyond One Loop Approximation

So far: Most phenomenological approaches to T-odd TMDs
→ Final state interactions modeled by a one-gluon exchange



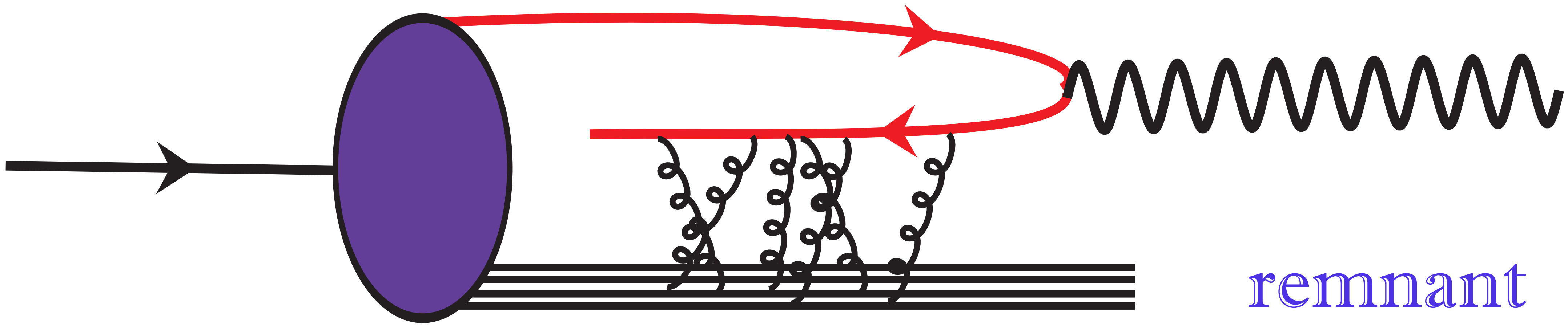
e.g. Diquark-model, MIT-Bag model etc.

Sivers-effect ~5%, $f_{1T}^{\perp,(1)u,d} \simeq \mp 0.05$

$\alpha_s \simeq 0.2 - 0.3$ "strength of FSI"

BHS (02)
Ji-Yuan (02)
LG, G. Goldstein (02,03..
LG, GRG, Schlegel (08)
Bacchetta,Conti,Radici et al. (08, 10)
Lu Schmidt (05, 06)

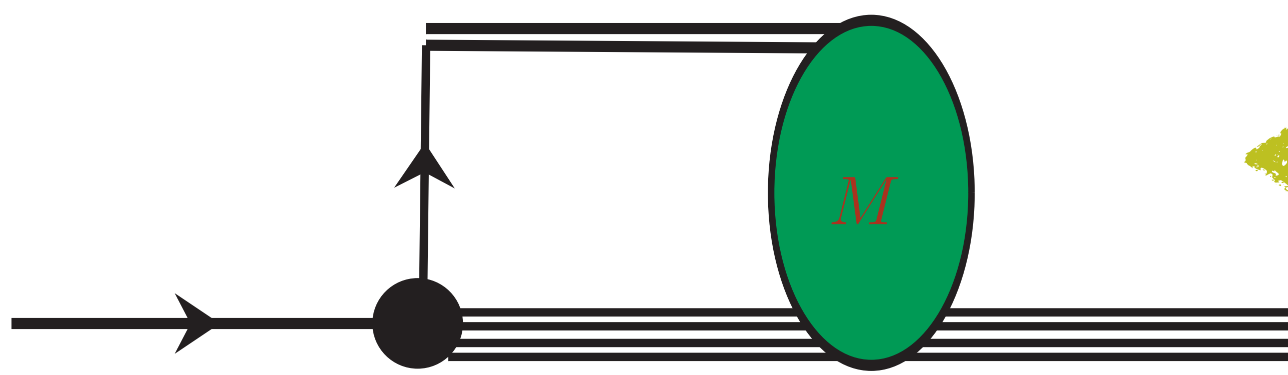
Explore non-pertb. FSIs-Links & Gauge Link

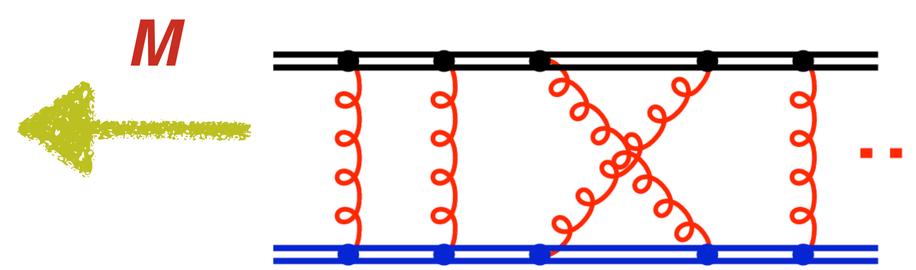


Non-perturbative calculation of FSIs

L.G. & Marc Schlegel
Phys.Lett.B685:95-103, 2010 &
Mod.Phys.Lett.A24:2960-2972,2009

$W\sim$





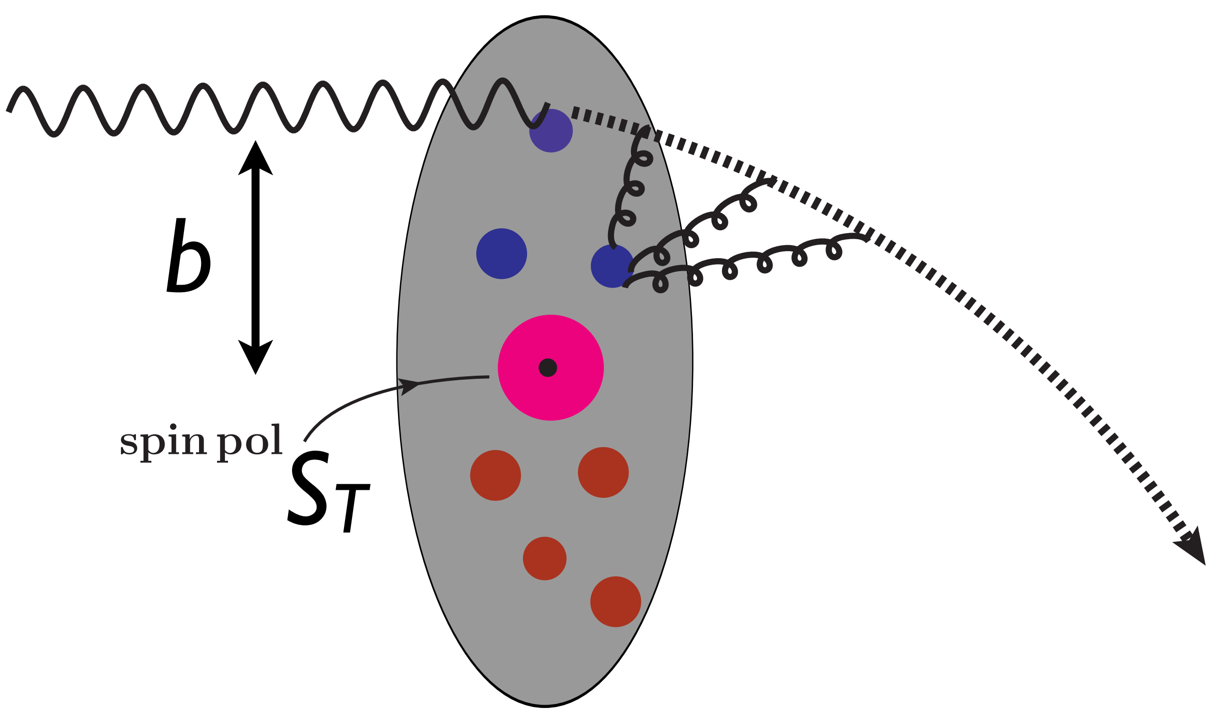
$$\epsilon_T^{ij} k_T^i S_T^j f_{1T}^{\perp}(x, \vec{k}_T^2) = -\frac{M}{8(2\pi)^3(1-x)P^+}\left(\bar{W}\gamma^+ W\Big|_{S_T} - \bar{W}\gamma^+ W\Big|_{-S_T}\right)$$

Fruitful to exploit 2+1 Dimension Transverse Structure and TSSAs and TMDs

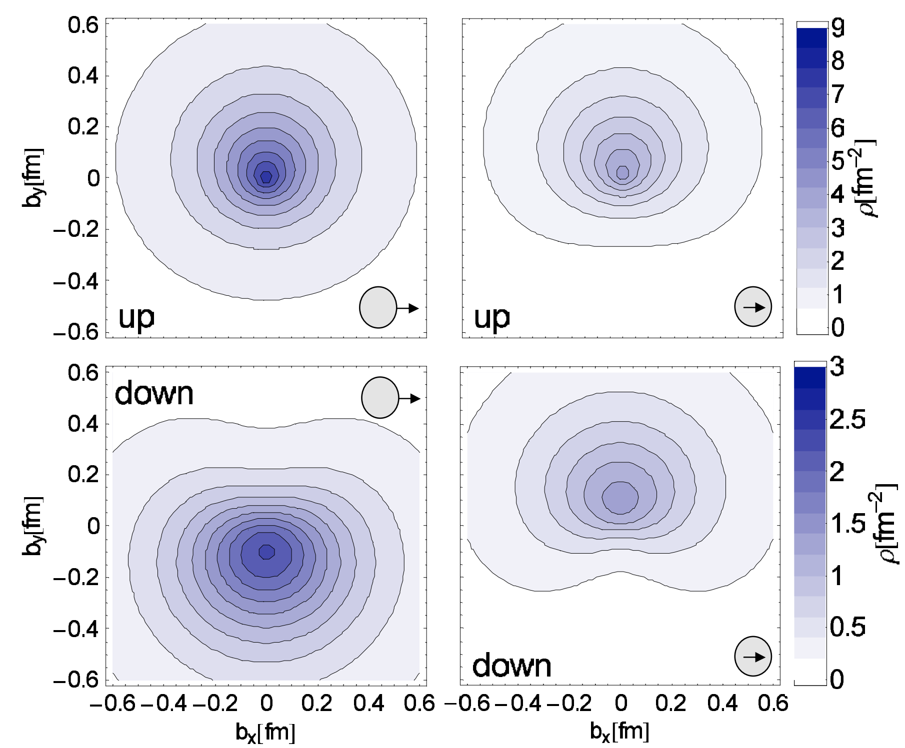
Intuitive picture of Sivers asymmetry: Spatial distortion in transverse plane due to polarization+ FSI leads to observable effect

Non-zero Left Right (Sivers) momentum asymmetry

M. Burkardt [Nucl.Phys. A735, 185], [PRD66, 114005]



Gockler et al. PRL07 x-moments of IP-GPDs



$$\vec{S} \cdot (\hat{P} \times \vec{k}_\perp)\, f_{1T}^{\perp}(x, \vec{k}_\perp^2)$$

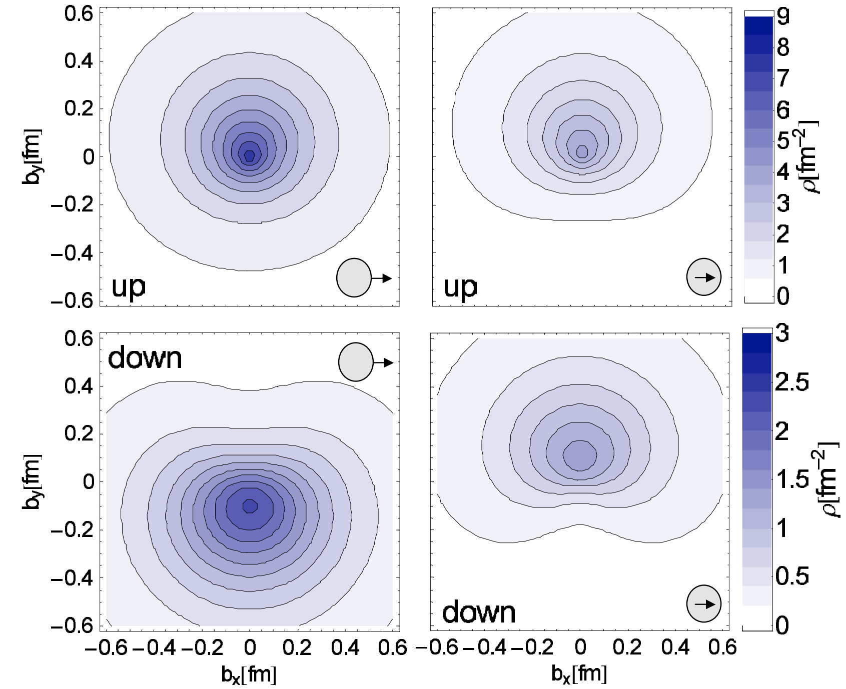
$$\vec{S} \cdot (\hat{P} \times \vec{b}) \left(\mathcal{E}(x, \vec{b}^2)\right)'$$

Used to predict sign of TSSA-Sivers

Burkardt 02,04 NPA PRD

$$d^y_q = \frac{1}{2M}\int dx \int d^2\mathbf{b}_\perp \mathcal{E}_q(x,\mathbf{b}_\perp)$$

$$= \frac{1}{2M}\int dx E_q(x,0,0) = \frac{F_{2,q}(0)}{2M} = \frac{\kappa}{2M}$$



$$\kappa^p = 1.79\,, \quad \kappa^n = -1.91$$

$$\longrightarrow \kappa^{u/p} = 1.67\,, \quad \kappa^{d/p} = -2.03$$

w/ attractive interactions

$$f_{1T}^{\perp(u)} = \text{neg} \quad \& \quad f_{1T}^{\perp(d)} = \text{pos}$$

Anselmino et al. PRD 05, EPJA 08

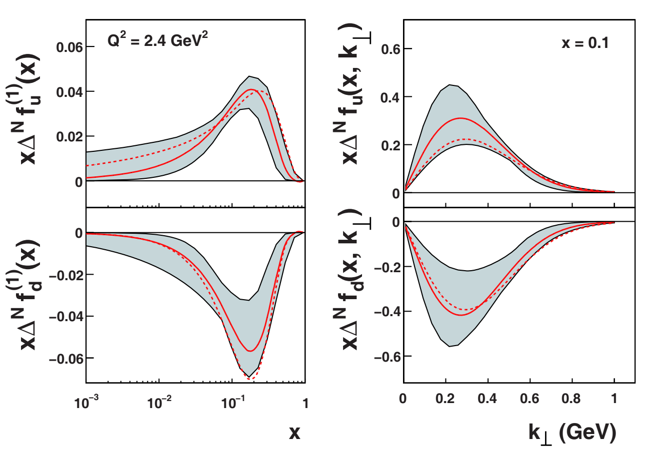


Fig. 7. The Sivers distribution functions for u and d flavours, at the scale $Q^2 = 2.4\,(\mathrm{GeV}/c)^2$, as determined by our present fit (solid lines), are compared with those of our previous fit [2] of SIDIS data (dashed lines), where π^0 and kaon productions were not considered and only valence quark contributions were taken into account. This plot clearly shows that the Sivers functions previously found are consistent, within the statistical uncertainty bands, with the Sivers functions presently obtained.

Sivers

Gamberg, Goldstein, Schlegel PRD 77, 2008

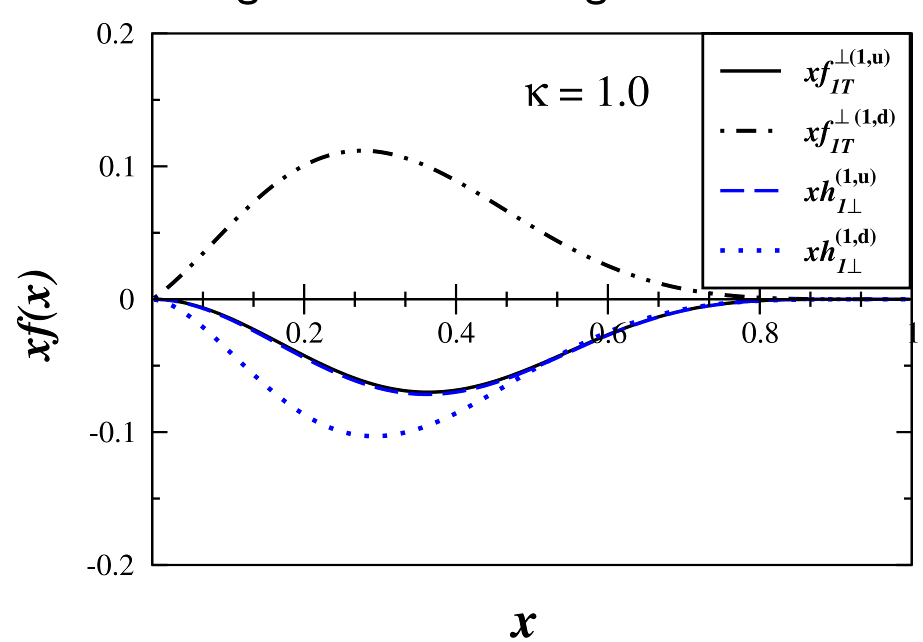


FIG. 5 (color online). The first moment of the Boer-Mulders and Sivers functions versus x for $\kappa = 1.0$.

"Spin-Orbit kinematics"

Analysis of correlators for TMDs and IP-GPDs similar forms

Burkhardt-02 PRD & ...
Diehl Hagler-05 EPJC,
Meissner, Metz, Goeke 07 PRD

$$\Phi^q(x,\vec{k}_T;S) = f_1^q(x,\vec{k}_T^2) - \frac{\epsilon_T^{ij} k_T^i S_T^j}{M} f_{1T}^{\perp q}(x,\vec{k}_T^2)\,,$$

$$\mathcal{F}^q(x,\vec{b}_T;S) = \mathcal{H}^q(x,\vec{b}_T^2) + \frac{\epsilon_T^{ij} b_T^i S_T^j}{M}\left(\mathcal{E}^q(x,\vec{b}_T^2)\right)'\,,$$

$\mathbf{k}_T \leftrightarrow \mathbf{b}_T$ Not conjugates (!) and ...

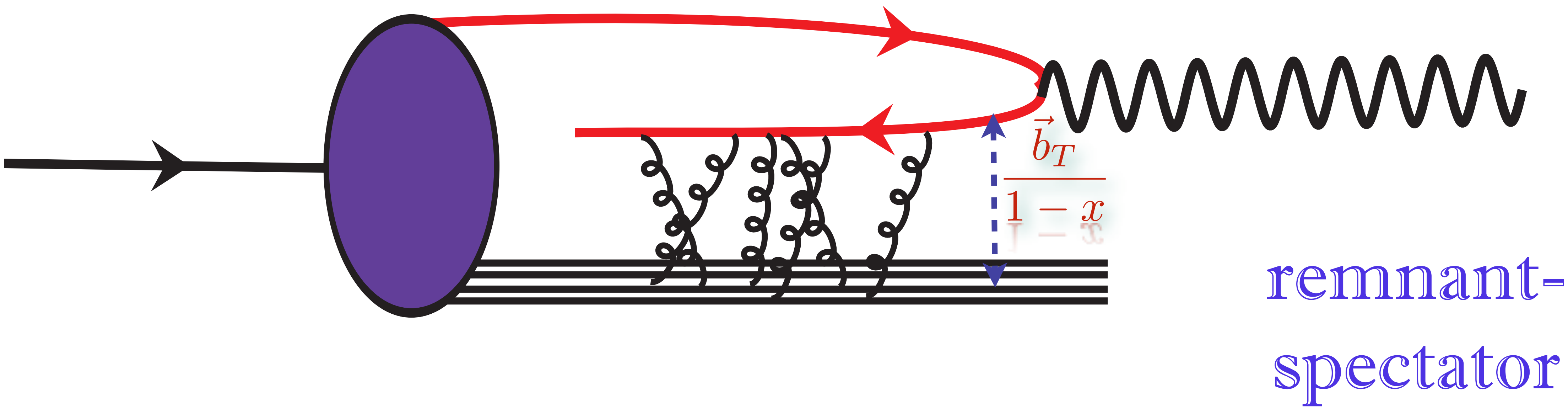
$f_{1T}^{\perp}(x,\vec{k}_T^2)$ "Naive T-odd"

$\left(\mathcal{E}(x,\vec{b}_T^2)\right)'$ "Naive T-even"

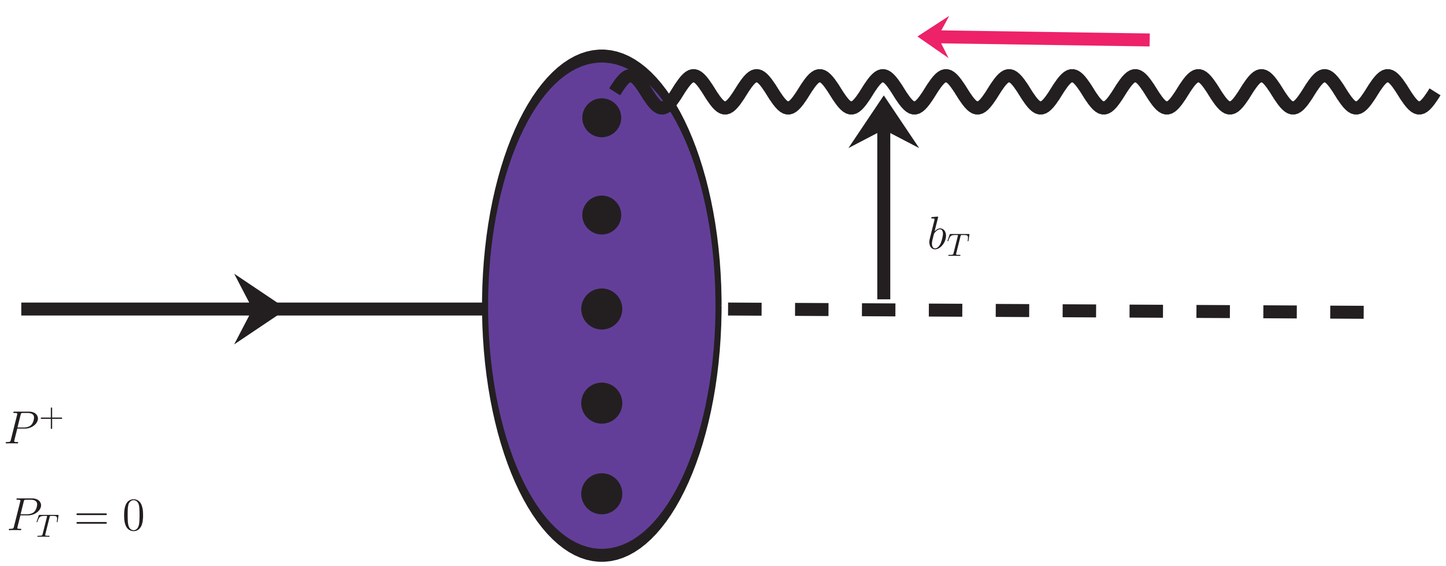
FSIs needed.... Burkardt PRD 02 & NPA 04

How do we test this further?

Summing Gauge Link, Impact parameter & spectator remnant



Fourier transform of GPD $F(x,0,\vec{\Delta}_T)$ @ $\xi = 0$



Burkardt PRD 00, 02, 04...

Localizing partons: impact parameter

- states with definite light-cone momentum p^+ and transverse position (impact parameter):

Soper PRD1977 $|p^+, \boldsymbol{b}\rangle = \int d^2\boldsymbol{p}\, e^{-i\boldsymbol{b}\boldsymbol{p}}\, |p^+, \boldsymbol{p}\rangle$

$$\mathcal{F}(x,\vec{b}) = \int \frac{dz^-}{(2\pi)^2} e^{ixP^+z^-} \langle P^+; \vec{0}_T | \bar{q}\,(z_1)\, \mathcal{W}(z_1,z_2) q\,(z_2)\, | P^+; \vec{0}_T \rangle$$

$$z_{1/2} = \pm \frac{z^-}{2} n_- + \frac{\vec{b}_T}{2}$$

$$\begin{aligned} \mathcal{F}(x,\vec{b}) &= \int \frac{d^2\Delta_T}{(2\pi)^2} e^{i\vec{\Delta}_T\cdot\vec{b}}\, F(x,0,\vec{\Delta}_T) \\ &= \mathcal{H}(x,\vec{b}) + \frac{\epsilon_T^{ij} b_T^i S_T^j}{M} \Big(\mathcal{E}(x,\vec{b})\Big)' \end{aligned}$$

F.T.

$$\vec{b} \leftrightarrow \vec{\Delta}_T$$

Prob. of finding unpol. quark w/ long momentum x at position b_T in trans. polarized S_T nucleon: spin independent $\mathcal{H}$ and spin flip part $\mathcal{E}'$

Observable to test this possible connection btnw TMD and Impact par. picture?

Gluonic Pole ME

$$\langle k_T^i \rangle_T(x) = \int d^2k_T\, k_T^i\, \tfrac{1}{2}\Big[\mathrm{Tr}[\gamma^+\Phi(\vec{S}_T)] - \mathrm{Tr}[\gamma^+\Phi](-\vec{S}_T)\Big]$$

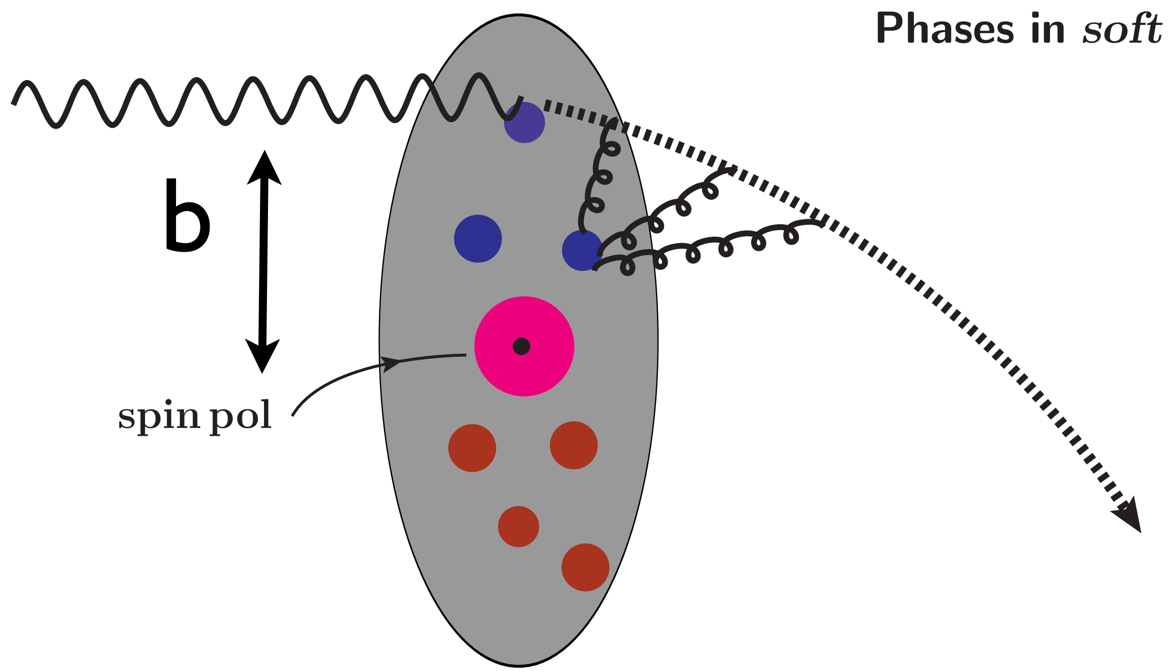
$$\langle k_T^i \rangle(x) = \int d^2b_T \int \frac{dz^-}{2(2\pi)} e^{ixP^+z^-} \boxed{\langle P^+; \vec{0}_T; S_T | \bar{\psi}(z_1)\gamma^+[z_1; z_2]} I^i(z_2) \boxed{\psi(z_2)|P^+; \vec{0}_T; S_T\rangle}$$

$$z_{1/2} = \mp\frac{z^-}{2} n_- + b_T$$

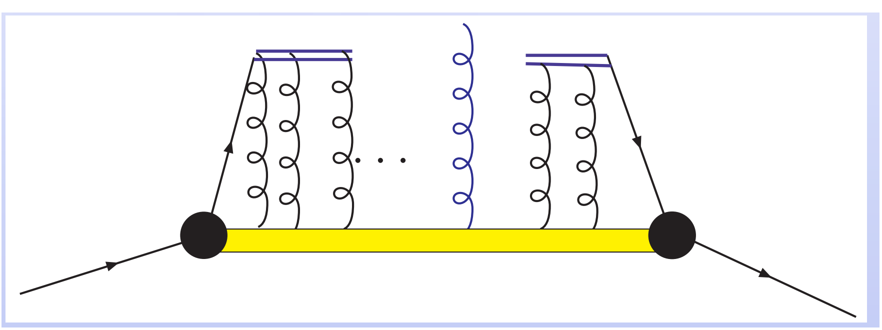
Impact parameter rep for GPD E

$$I^i(z^-) = \int dy^-[z^-; y^-] gF^{+i}(y^-)[y^-; z^-]$$

Soft gluonic pole op



Phases in *soft* poles of propagator in hard subprocess Efremov & Teryaev :PLB 1982



Note on Distortion and FSIs

M. Burkardt [Nucl.Phys. A735, 185], [PRD66, 114005]

$$\langle k_T^{q,i}(x)\rangle_{UT} = \int d^2k_T\, k_T^i\, \tfrac{1}{2}\left[\mathrm{Tr}[\gamma^+\Phi(\vec{S}_T)] - \mathrm{Tr}[\gamma^+\Phi](-\vec{S}_T)\right]$$

Manipulate gauge link and trnsfm to $\vec{b}$ space

1) $\langle k_T^{q,i}(x)\rangle_{UT} = \frac{1}{2}\int d^2\vec{b}_T \int \frac{dz^-}{2\pi} e^{ixP^+z^-}\langle P^+,\vec{0}_T;S|\bar{\psi}(z_1)\gamma^+\mathcal{W}(z_1;z_2)I^{q,i}(z_2)\psi(z_2)|P^+,\vec{0}_T;S\rangle$

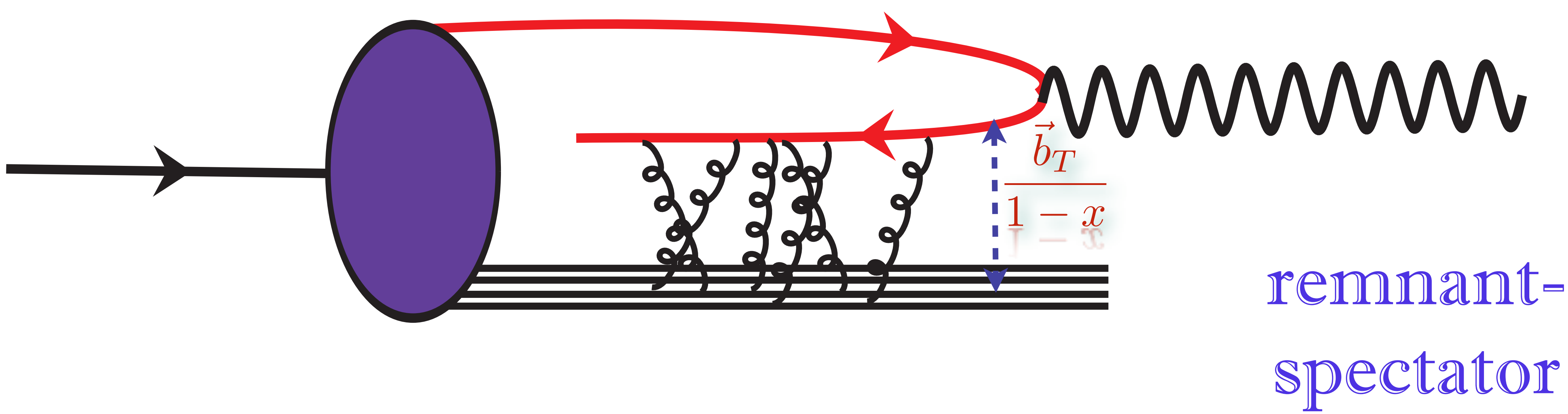
2) $\mathcal{F}^{q[\Gamma]}(x,\vec{b}_T;S) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-}\langle P^+,\vec{0}_T;S|\bar{\psi}(z_1)\Gamma\mathcal{W}(z_1;z_2)\psi(z_2)|P^+,\vec{0}_T;S\rangle$, $\Gamma \equiv \gamma^+$

Comparing expressions difference is additional factor,
$I^{q,i}$ and integration over $\vec{b}$

Conjecture: factorization FSI and spatial distortion:

$$\langle k_T^i \rangle(x) = M \epsilon_T^{ij} S_T^i f_{1T}^{\perp(1)} \approx \int d^2 b_T \mathcal{I}^i(x, \vec{b}_T^2) \frac{\vec{b}_T \times \vec{S}_T}{M} \frac{\partial}{\partial b_T^2} \mathcal{E}(x, \vec{b}_T^2)$$

$\mathcal{I}^i(x, \vec{b}_T^2)$ Lensing Function



Boer Mulders as well ...

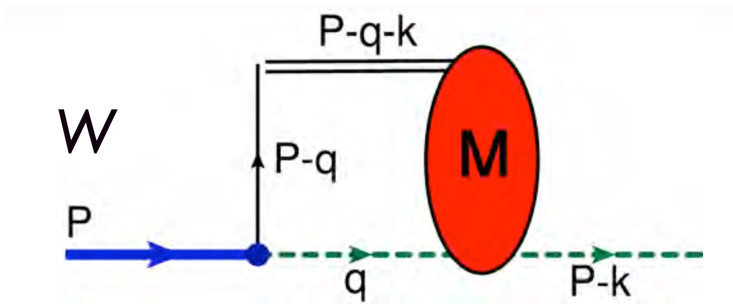
- Av. transv. momentum of transv. pol. partons in an unpol. hadron:

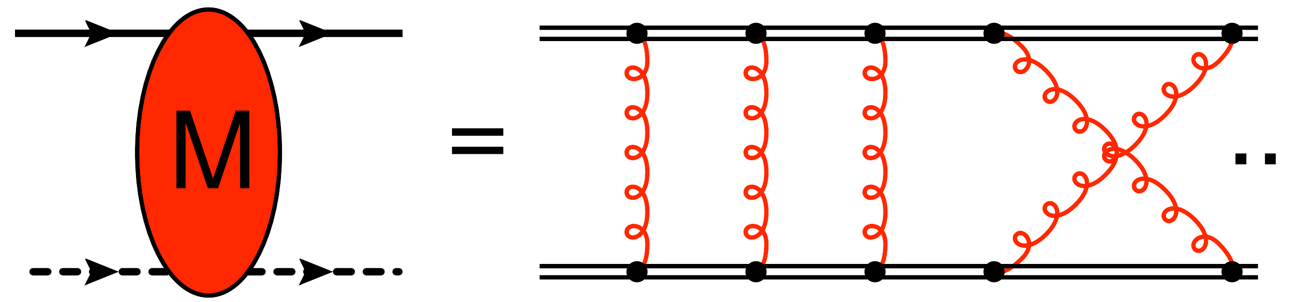
$$\boxed{\langle k_T^i \rangle^j(x) \;=\; \int d^2k_T\, k_T^i\, \tfrac{1}{2}\Big(\Phi^{[i\sigma^{i+}\gamma^5]}(S) + \Phi^{[i\sigma^{i+}\gamma^5]}(-S)\Big)}$$

$$\Rightarrow \quad \boxed{-2M^2\, h_1^{\perp,(1)}(x) \simeq \int d^2b_T\, \vec{b_T} \cdot \vec{\mathcal{I}}(x, \vec{b}_T)\, \frac{\partial}{\partial b_T^2}\Big(\mathcal{E}_T + 2\tilde{\mathcal{H}}_T\Big)(x, \vec{b}_T^2)}$$

Diehl & Hagler EJPC (05), Burkardt PRD (04)

- **Relativistic Eikonal models: Treat FSI non-perturbatively.**





We calc "W" again....

$$\epsilon_T^{ij} k_T^i S_T^j f_{1T}^{\perp}(x, \vec{k}_T^2) = -\frac{M}{8(2\pi)^3(1-x)P^+}\left(\bar{W}\gamma^+ W\Big|_{S_T} - \bar{W}\gamma^+ W\Big|_{-S_T}\right)$$

$$\Delta W(P,k) = \boxed{\int \frac{d^4q}{(2\pi)^4} g_N[(P-q)^2] \frac{[(\not{P} - \not{q} + m_q)u(P,S)]_i \mathcal{M}_{bc}^{ab}(q, P-k)}{[n\cdot(P-k-q)+i\varepsilon][(P-q)^2+m_q^2+i\varepsilon][q^2-m_s^2+i\varepsilon]}}$$

- **Step 1: Integration over q^-:**

Assume no q^- & q^+ poles in M.

q^- - poles at one loop for higher twist T-odd TMDs [Gamberg, Hwang, Metz, MS, PLB 639, 508]

- **Step 2: Integration over q^+:**

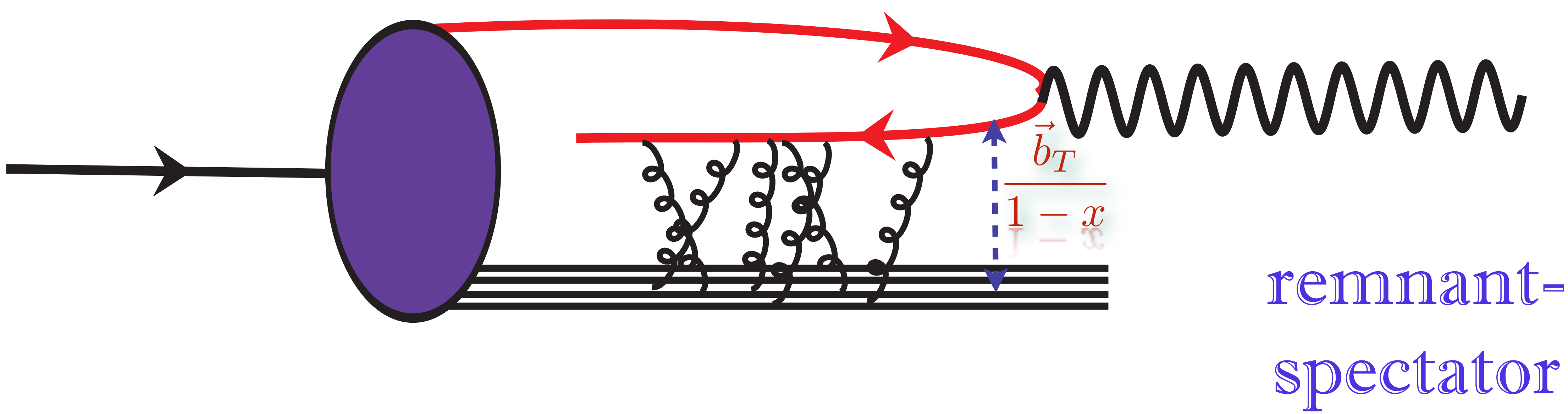
$$\frac{1}{(1-x)P^+ - q^+ + i\varepsilon} = \mathrm{P}\frac{1}{(1-x)P^+ - q^+} - i\pi\delta((1-x)P^+ - q^+)$$

Fix the q+ - pole
emphasizes a "natural" picture of FSI
equivalent to Cutkosky cut, assumptions of Step 1 valid in Eikonal models

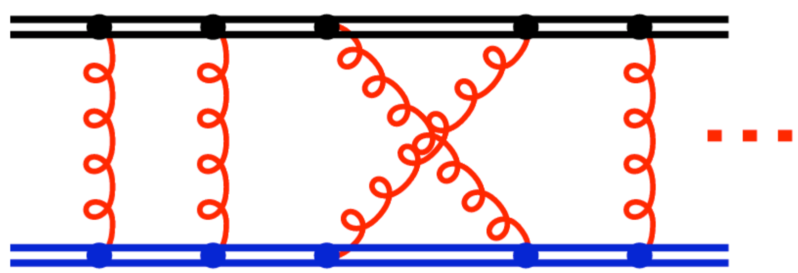
Conjecture born out factorization FSI and spatial distortion in eikonal + spectator approximation

$$\langle k_T^i \rangle(x) = M \epsilon_T^{ij} S_T^i f_{1T}^{\perp(1)} \approx \int d^2 b_T \mathcal{I}^i(x, \vec{b}_T^2) \frac{\vec{b}_T \times \vec{S}_T}{M} \frac{\partial}{\partial b_T^2} \mathcal{E}(x, \vec{b}_T^2)$$

$\mathcal{I}^i(x, \vec{b}_T^2)$ Lensing Function



Eikonal Color calculation and path ordered gauge link



Abarabanel Itzykson PRL 69
Gamberg Milton PRD 1999
Fried et al. 2000

$$G^{ab}_{\mathrm{eik}}(x,y|A) = -i\int_0^\infty ds\, \mathrm{e}^{-is(m_q - i0)}\delta^{(4)}(x-y-sv)\left(\mathrm{e}^{-ig\int_0^s d\beta\, v\cdot A^\alpha(y+\beta v)\, t^\alpha}\right)^{ab}_+$$

Trick to disentangle the A-field and the color matrices t: Functional FT

$$\left(\mathrm{e}^{-ig\int_0^s d\beta\, v\cdot A^\alpha(y+\beta v)\, t^\alpha}\right)^{ab}_+ = \mathcal{N}'\int\mathcal{D}\alpha\int\mathcal{D}u\, \mathrm{e}^{i\int d\tau\, \alpha^\beta(\tau)u^\beta(\tau)}\mathrm{e}^{ig\int d\tau\, \alpha^\beta(\tau)\, v\cdot A^\beta(y+\tau v)}\left(\mathrm{e}^{i\int_0^s d\tau\, t^\beta u^\beta(\tau)}\right)^{ab}_+$$

FLOW CHART for calculation of Boer Mulders

L.G. & Marc Schlegel

Phys.Lett.B685:95-103,2010 & Mod.Phys.Lett.A24:2960-2972,2009.

$$2m_\pi^2 h_1^{\perp(1)}(x) \simeq \int d^2 b_T\, \vec{b}_T \cdot \vec{\mathcal{I}}(x,\vec{b}_T)\, \frac{\partial}{\partial \vec{b}_T^2} \mathcal{H}_1^\pi(x,\vec{b}_T^2),$$

$$I^i(x,\vec{q}_T) = \frac{1}{N_c}\int \frac{d^2 p_T}{(2\pi)^2}(2p_T - q_T)^i \left(\Im[\bar{\mathrm{M}}^{\mathrm{eik}}]\right)_{\delta\beta}^{\alpha\delta}(|\vec{p}_T|)$$
$$\left((2\pi)^2\delta^{\alpha\beta}\delta^{(2)}(\vec{p}_T-\vec{q}_T) + \left(\Re[\bar{\mathrm{M}}^{\mathrm{eik}}]\right)_{\gamma\alpha}^{\beta\gamma}(|\vec{p}_T-\vec{q}_T|)\right).$$

Non-pertb FSIs in here

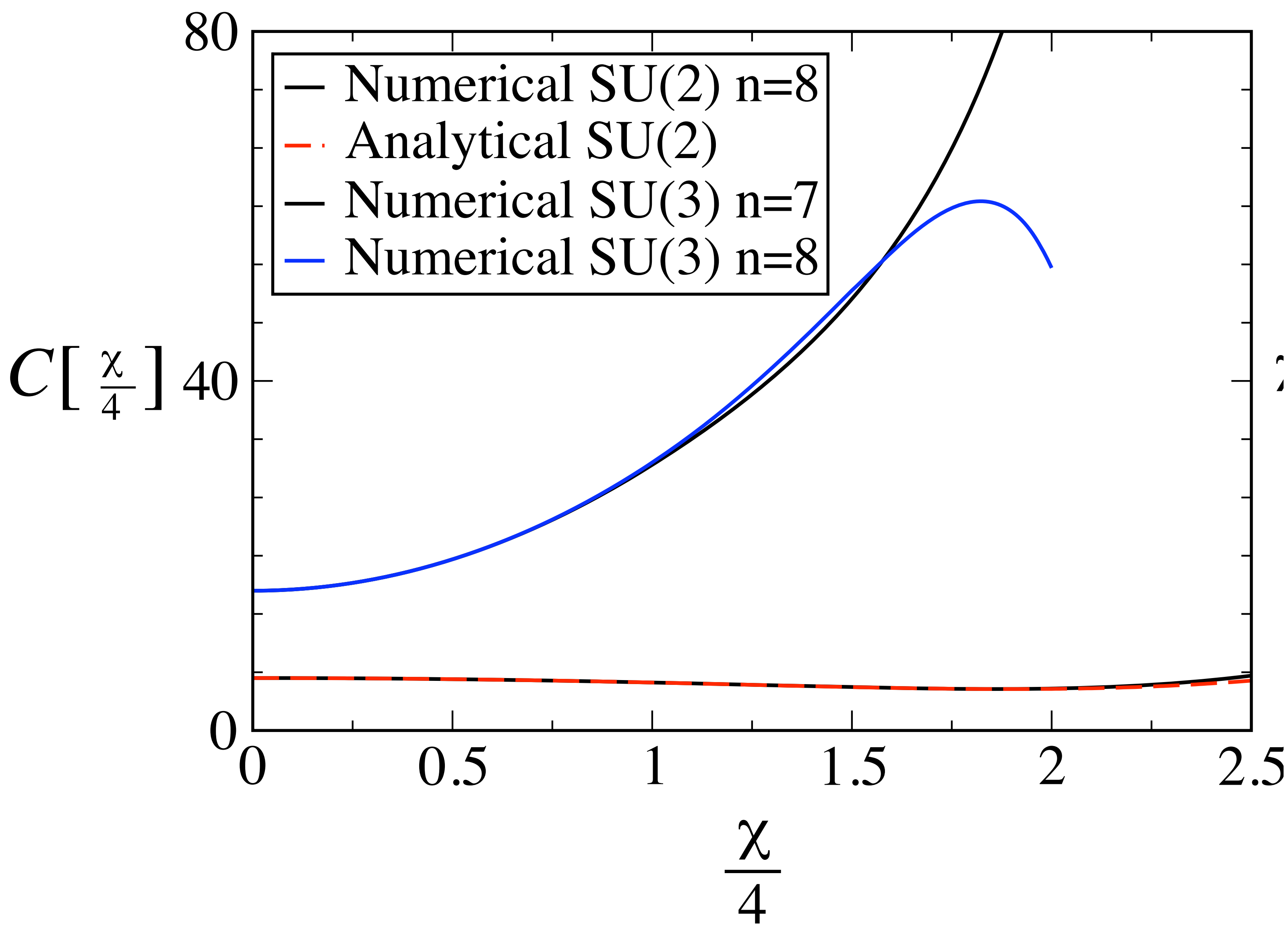
$$\left(\mathrm{M}^{\mathrm{eik}}\right)_{\delta\beta}^{\alpha\delta}(x,|\vec{q}_T+\vec{k}_T|) = \frac{(1-x)P^+}{m_s}\int d^2 z_T\, \mathrm{e}^{-i\vec{z}_T\cdot(\vec{q}_T+\vec{k}_T)} \quad (20)$$
$$\times\left[\int d^{N_c^2-1}\alpha \int \frac{d^{N_c^2-1}u}{(2\pi)^{N_c^2-1}}\,\mathrm{e}^{-i\alpha\cdot u}\left(\mathrm{e}^{i\chi(|\vec{z}_T|)t\cdot\alpha}\right)_{\alpha\delta}\left(\mathrm{e}^{it\cdot u}\right)_{\delta\beta} - \delta_{\alpha\beta}\right].$$

COLOR Integral

$$f_{\alpha\beta}(\chi) \equiv \int d^{N_c^2-1}\alpha \int \frac{d^{N_c^2-1}u}{(2\pi)^{N_c^2-1}}\,\mathrm{e}^{-i\alpha\cdot u}\left(\mathrm{e}^{i\chi(|\vec{z}_T|)t\cdot\alpha}\right)_{\alpha\delta}\left(\mathrm{e}^{it\cdot u}\right)_{\delta\beta} - \delta_{\alpha\beta}$$

Lensing Function & untangling the COLOR FACTOR

$$\mathcal{I}^i(x,\vec{b}_T) = \frac{(1-x)}{2N_c}\frac{b_T^i}{|\vec{b}_T|}\frac{\chi'}{4}C\left[\frac{\chi}{4}\right],$$



$$f_{\alpha\beta}(\chi) = \sum_{n=1}^{\infty}\frac{(i\chi)^n}{(n!)^2}\sum_{a_1=1}^{N_c^2-1}\cdots\sum_{a_n=1}^{N_c^2-1}\sum_{P_n}(t^{a_1}\ldots t^{a_n}t^{a_{P_n(1)}}\ldots t^{a_{P_n(n)}})_{\alpha\beta}\,.$$

Eikonal Phase and FSIs

$$\chi^{DS}(|\vec{z}_T|) = 2\int_0^\infty dk_T\, k_T \alpha_s(k_T^2) J_0(|\vec{z}_T| k_T) Z(k_T^2, \Lambda_{QCD}^2)/k_T^2.$$

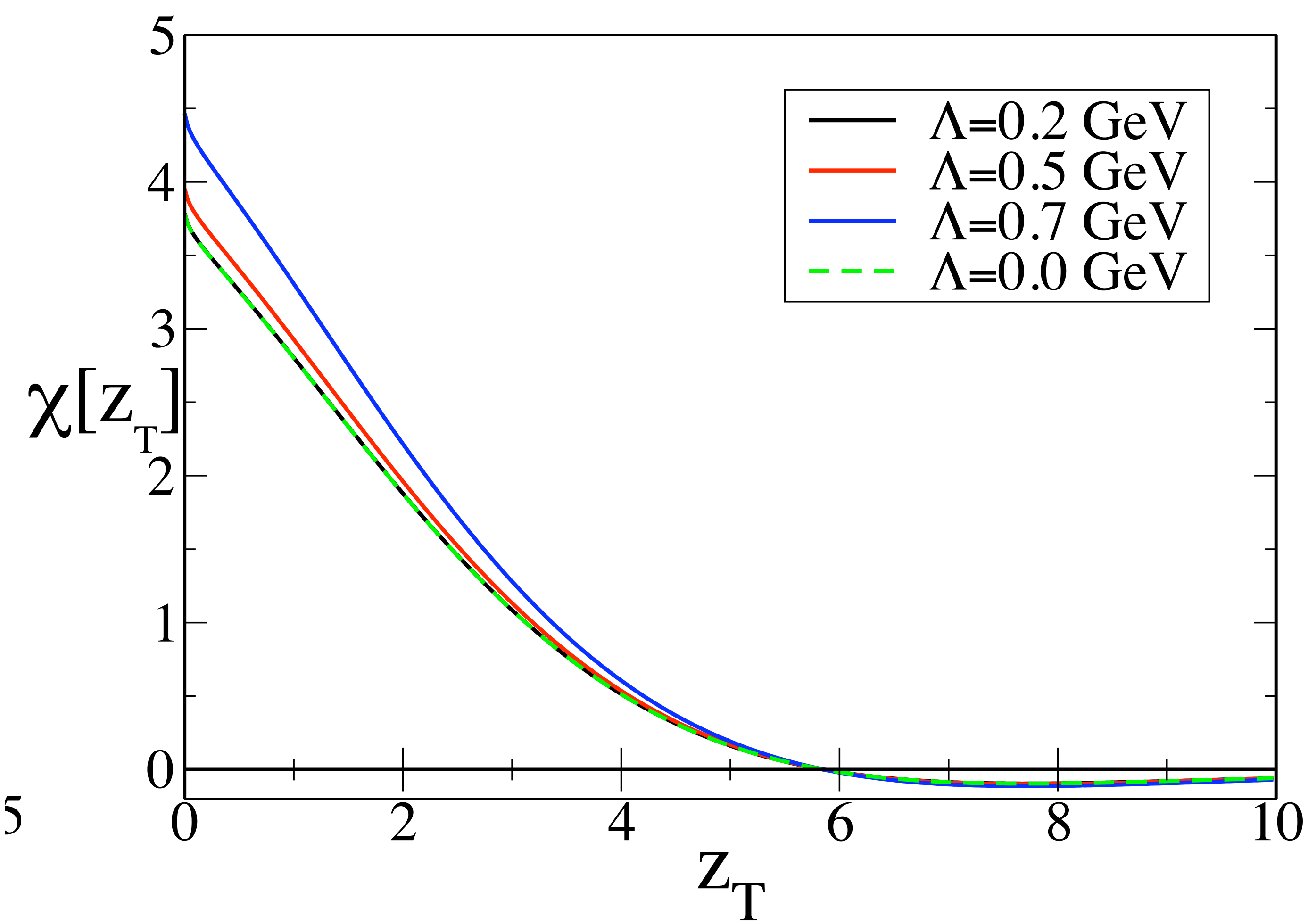
$$\alpha_s(\mu^2) = \frac{\alpha_s(0)}{\ln\left[\mathrm{e} + \mathrm{a}_1(\mu^2/\Lambda^2)^{\mathrm{a}_2} + \mathrm{b}_1(\mu^2/\Lambda^2)^{\mathrm{b}_2}\right]}. \qquad (35)$$

The values for the fit parameters are $\Lambda = 0.71$ GeV, $a_1 = 1.106$, $a_2 = 2.324$, $b_1 = 0.004$ and $b_2 = 3.169$.

$$\begin{aligned} Z(p^2,\mu^2) &= p^2 \mathcal{D}^{-1}(p^2,\mu^2) \\ &= \left(\frac{\alpha_s(p^2)}{\alpha_s(\mu^2)}\right)^{1+2\delta} \left(\frac{c\left(\frac{p^2}{\Lambda^2}\right)^{\kappa} + d\left(\frac{p^2}{\Lambda^2}\right)^{2\kappa}}{1 + c\left(\frac{p^2}{\Lambda^2}\right)^{\kappa} + d\left(\frac{p^2}{\Lambda^2}\right)^{2\kappa}}\right)^2, \end{aligned} \qquad (36)$$

Fisher & Alkofer prd 03, Annal of phys. 09

with the parameters $c = 1.269$, $d = 2.105$, and $\delta = -\frac{9}{44}$.



5

- use running coupling extended to non-perturbative regime
- gluon non-perturbative gluon propagator

Lensing Function

Express Lensing Function in terms of Eikonal Phase:

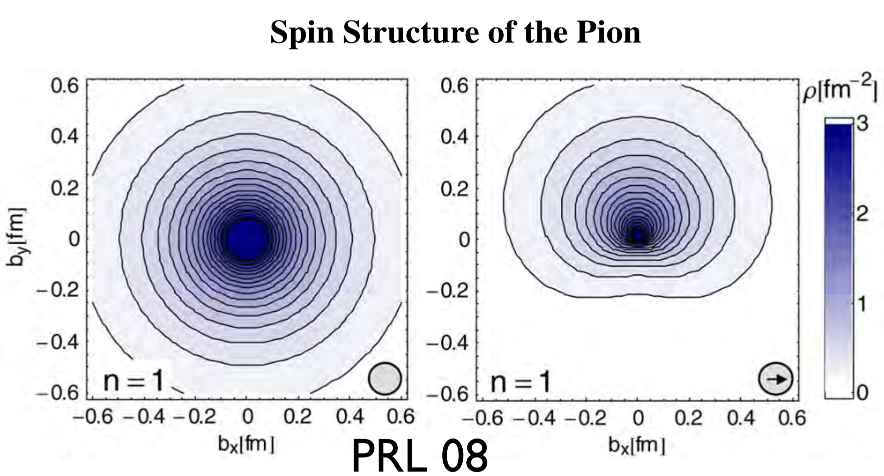
$$\boxed{\mathcal{I}^i_{(N=1)}(x,\vec{b}_T)=\frac{1}{4}\frac{b^i_T}{|\vec{b}_T|}\chi'\big(\tfrac{|\vec{b}_T|}{1-x}\big)\Big[1+\cos\chi\big(\tfrac{|\vec{b}_T|}{1-x}\big)\Big]}$$

$$\boxed{\mathcal{I}^i_{(N=2)}(x,\vec{b}_T)=\frac{1}{8}\frac{b^i_T}{|\vec{b}_T|}\chi'\big(\tfrac{|\vec{b}_T|}{1-x}\big)\Big[3(1+\cos\tfrac{\chi}{4})+\big(\tfrac{\chi}{4}\big)^2-\sin\tfrac{\chi}{4}\big(\tfrac{\chi}{4}-\sin\tfrac{\chi}{4}\big)\Big]\big(\tfrac{|\vec{b}_T|}{1-x}\big)}$$

$$\boxed{\mathcal{I}^i_{(N=3)}(x,\vec{b}_T)=\text{numerics}}$$

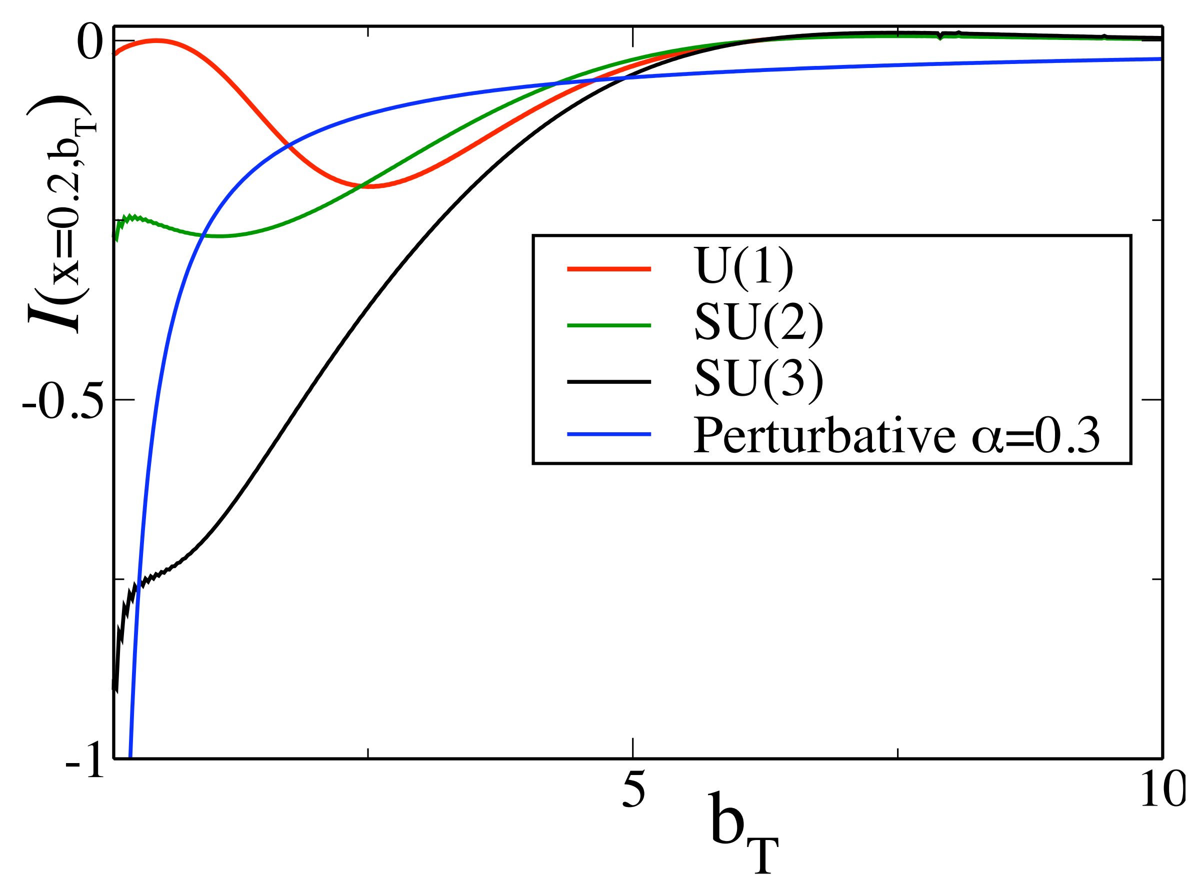
L.G. & Marc Schlegel

Phys.Lett.B685:95-103,2010 & Mod.Phys.Lett.A24:2960-2972,2009.



D. Brömmel,[1,2] M. Diehl,[1] M. Göckeler,[2] Ph. Hägler,[3]

FSI + distortion

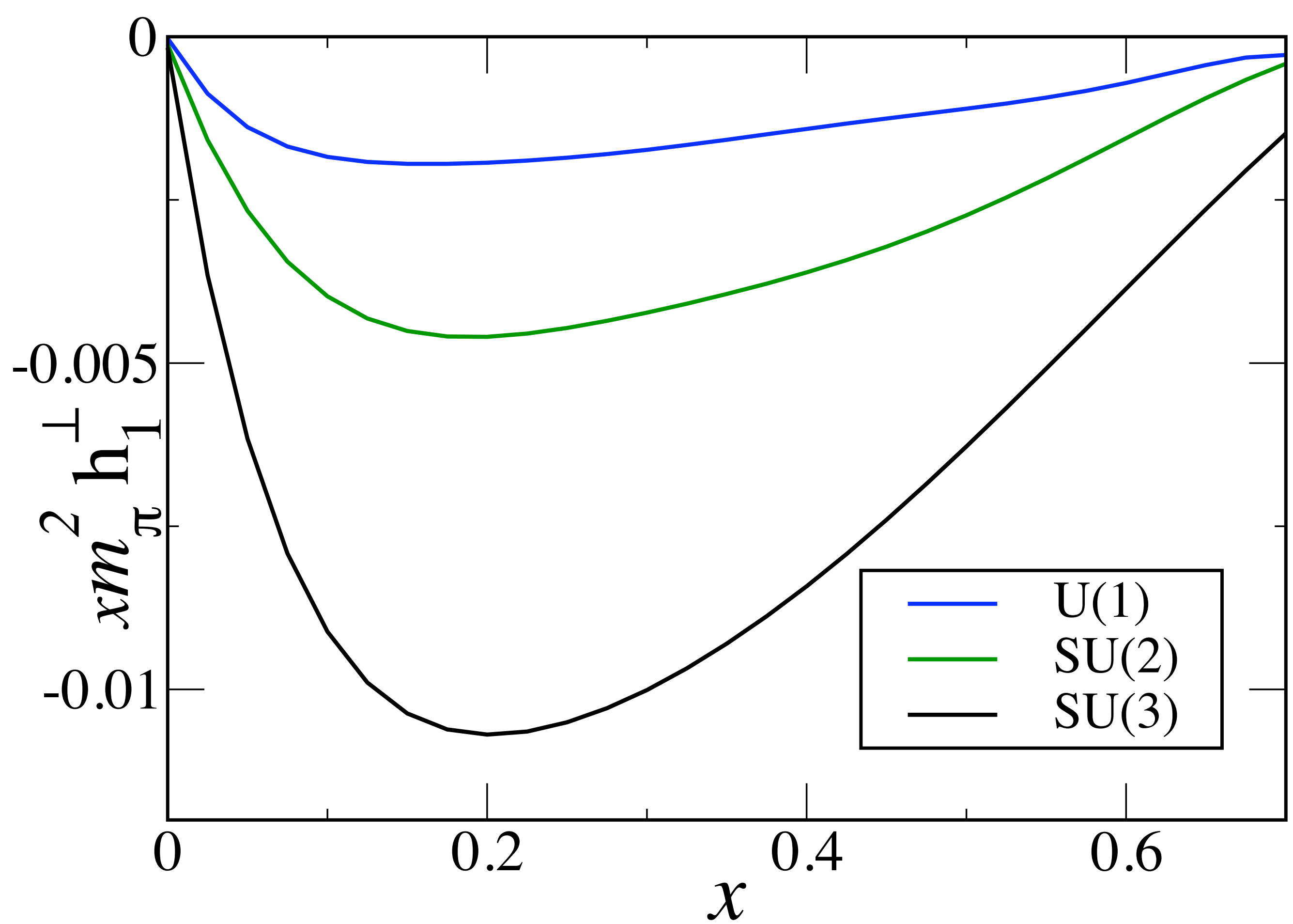


FSIs are negative and "grow" with Color!

Prediction for Boer-Mulders Function of PION

L.G. & Marc Schlegel

Phys.Lett.B685:95-103,2010 & Mod.Phys.Lett.A24:2960-2972,2009.

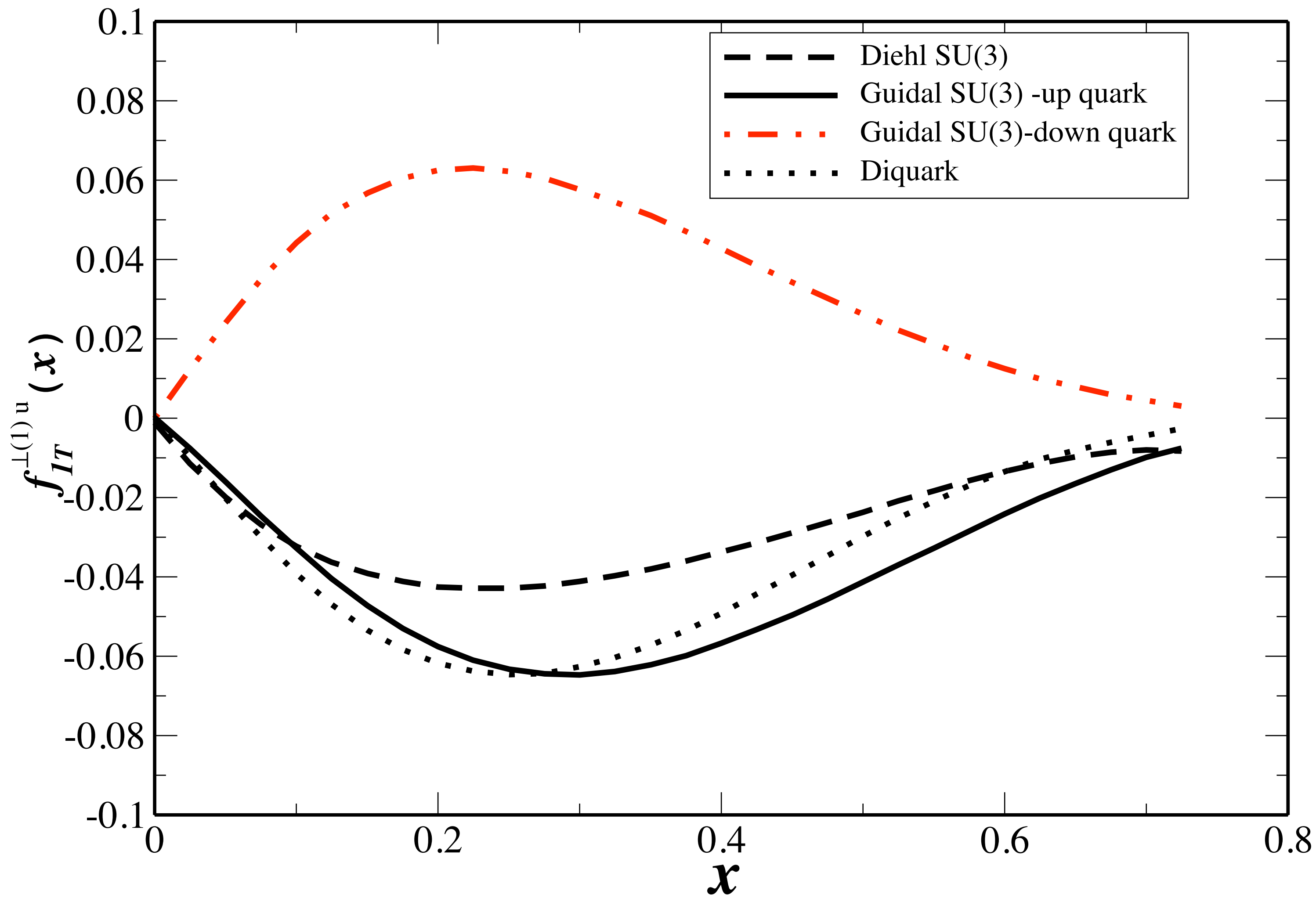


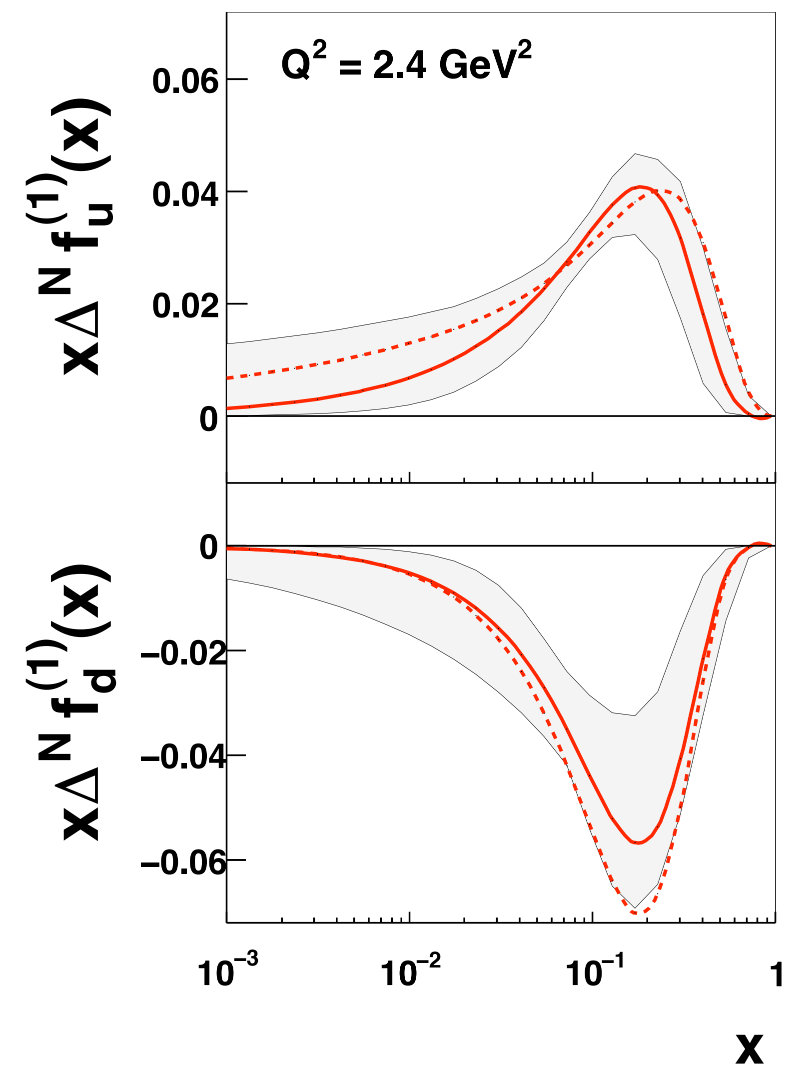
Relations produce a BM funct. approx equiv. to Sivers from HERMES

Expected sign i.e. FSI are negative

Answer will come from pion BM from COMPASS πN **Drell Yan**

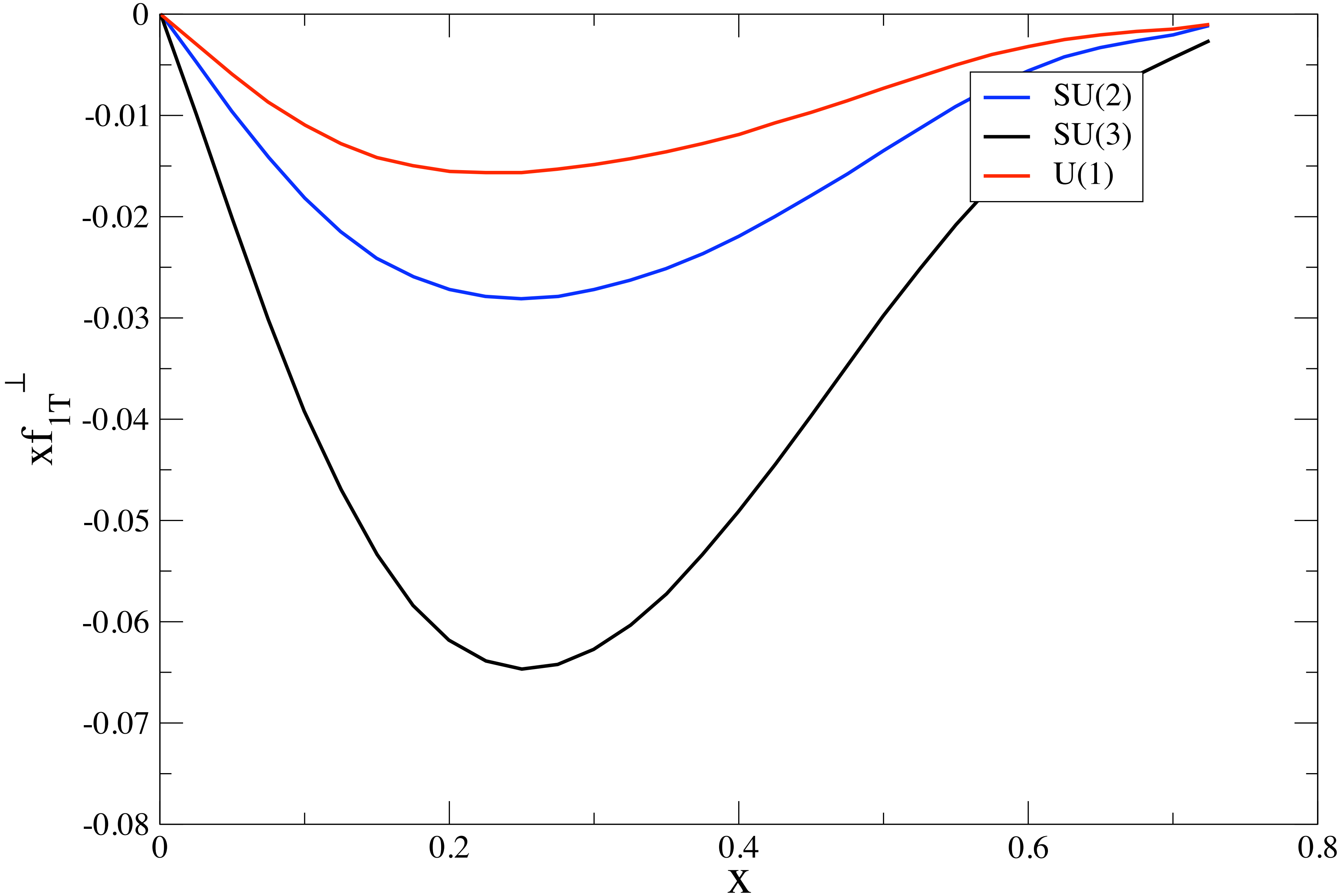
Results for u & d-quark Sivers





- Relations produce a Sivers effect 0.10-0.65 Nc=1 to 3
- Torino extraction ~ 0.05 SU(3) ! agrees with Chromodynamic LENSING
- Sivers effect increases with color
- Color tracing gives result of N_c counting of Pobylitsa however there are subleading contributions that are non-trivial in performing color tracing

Sivers function increases with color



Reality Check

Parm. of GTMD correlator hermiticity parity time-reversal

from Andreas Metz INT talk

$$(x, \xi, \vec{k}_T, \vec{\Delta}_T)$$

$$W^q = \frac{1}{2} \int \frac{dz^-}{2\pi} \frac{d^2\vec{z}_T}{(2\pi)^2} e^{ik\cdot z} \langle p'; \lambda' | \bar{\psi}\Big(-\frac{z}{2}\Big) \gamma^+ \mathcal{W}_{GTMD} \, \psi\Big(\frac{z}{2}\Big) |p; \lambda\rangle \Big|_{z^+=0}$$

- Projection onto GPDs and TMDs

$$\begin{aligned} F^q &= \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle p'; \lambda' | \bar{\psi}\Big(-\frac{z}{2}\Big) \gamma^+ \mathcal{W}_{GPD} \, \psi\Big(\frac{z}{2}\Big) |p; \lambda\rangle \Big|_{z^+=z_T=0} \\ &= \int d^2\vec{k}_T \, W^q \end{aligned}$$

$$\begin{aligned} \Phi^q &= \frac{1}{2} \int \frac{dz^-}{2\pi} \frac{d^2\vec{z}_T}{(2\pi)^2} e^{ik\cdot z} \langle p; \lambda' | \bar{\psi}\Big(-\frac{z}{2}\Big) \gamma^+ \mathcal{W}_{TMD} \, \psi\Big(\frac{z}{2}\Big) |p; \lambda\rangle \Big|_{z^+=0} \\ &= W^q \Big|_{\Delta=0} \end{aligned}$$

GTMD-Wigner Function Correlator

- Parameterization of GTMD-correlator **Miessner Metz & Schlegel JHEP 2008 & 2009**

  Example:

$$W^{q\,[\gamma^+]} = \frac{1}{2M}\,\bar{u}(p',\lambda')\left[F_{1,1} + \frac{i\sigma^{i+}k_T^i}{P^+}\,F_{1,2} + \frac{i\sigma^{i+}\Delta_T^i}{P^+}\,F_{1,3} + \frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}\,F_{1,4}\right]u(p,\lambda)$$

  $\longrightarrow$ GTMDs are complex functions: $F_{1,n} = F_{1,n}^e + i F_{1,n}^o$

- Implications for potential nontrivial relations
  - Relations of second type

$$E(x,0,\vec{\Delta}_T^2) = \int d^2\vec{k}_T\left[-F_{1,1}^e + 2\left(\frac{\vec{k}_T\cdot\vec{\Delta}_T}{\vec{\Delta}_T^2}\,F_{1,2}^e + F_{1,3}^e\right)\right]$$

$$f_{1T}^{\perp}(x,\vec{k}_T^2) = -F_{1,2}^o(x,0,\vec{k}_T^2,0,0)$$

$\longrightarrow$ No model-independent nontrivial relation between E and $f_{1T}^{\perp}$ possible

$\longrightarrow$ Relation in spectator model due to simplicity of the model

$\longrightarrow$ No information on numerical violation of relation

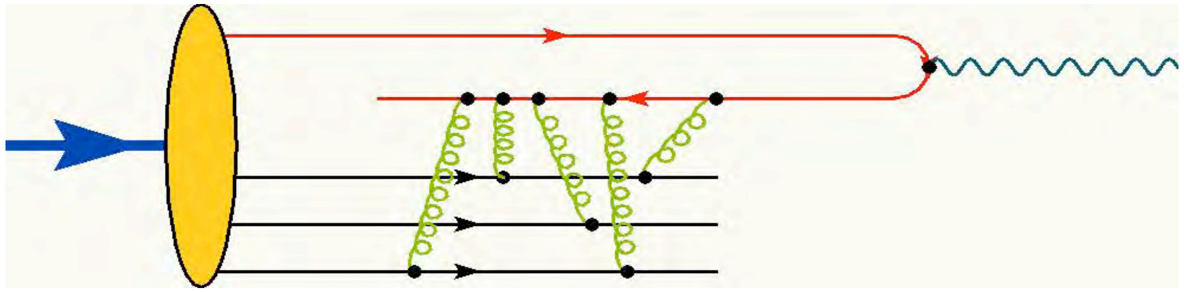
$\longrightarrow$ Likewise for nontrivial relation involving $h_1^{\perp}$

Conclusions

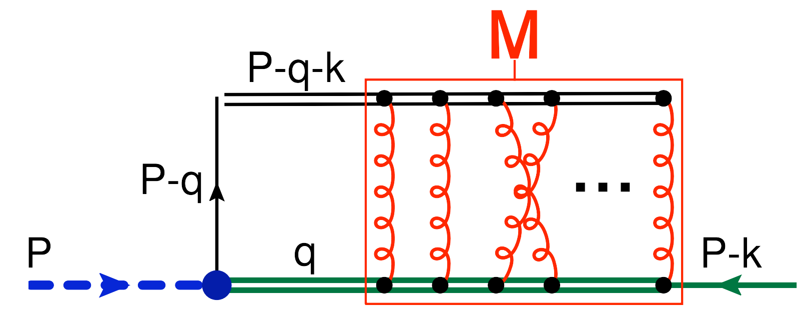
- **Going beyond one loop in spectator framework transverse distortion of T-odd TMDs from FSIs as path ordered Gauge link, factorize into Lensing function times transverse distortion**
- *Approximate dynamical relation good for phenomenological approach for model builders*
- **Pheno-Transverse Structure TMDs and TSSAs *b* and *k* asymm. An improved dynamical approach for FSI & model building**

"QCD calc" FSIs Gauge Links-Color Gauge Inv. "T-odd" TMDs

Calculation of M



- Calculate the amplitude M in a relativistice eikonal model:
  [1970's: Fried, Quiros, Levy, Sucher, Zuber, etc....]



Exact 4-point function for quark-diquark scattering:

$$T = -\mathrm{e}^{-iL_{12}}\left[\left(\mathrm{e}^{-\frac{i}{2}L_{11}}\mathcal{G}^{-1}(x_2,x_1|\bar{A}_1)\underbrace{\mathrm{e}^{\frac{1}{2}\mathrm{Tr}\ln\mathcal{G}(\bar{A}_1)}}_{\text{neglect}}\right)\times\left(\mathrm{e}^{-\frac{i}{2}L_{22}}\mathcal{K}^{-1}(y_2,y_1|\bar{A}_2)\underbrace{\mathrm{e}^{-\frac{1}{2}\mathrm{Tr}\ln\mathcal{K}(\bar{A}_2)}}_{\text{neglect}}\right)\right]\Bigg|_{\bar{A}_1=\bar{A}_2=0}$$

Linkage operator: $$\boxed{L_{ij} = -\int d^4z_1 d^4z_2 \frac{\delta}{\delta\bar{A}_i(z_1)}\mathcal{D}^{-1}(Z_1 - z_2)\frac{\delta}{\delta\bar{A}_j(z_2)}}$$

Eikonal approximation: $$\boxed{L_{ii}\mathcal{G}^{-1} = 0}$$

TMDs & Impact GPDs Project from GTMDs

$$W^{[\Gamma]}_{\lambda,\lambda'}(P,x,\mathbf{k}_T,\Delta,n) = \int dk^- \, W^{[\Gamma]}_{\lambda,\lambda'}(P,k,\Delta,n)$$

Integ. small component,

GTMD--Meissner Metz Schlegel, 07

$\int d^2\mathbf{k}_T$

$\Delta = 0$

$$F^{[\Gamma]}_{\lambda,\lambda'}(x,\xi,t) = \int d^2\mathbf{k}_T \, W^{[\Gamma]}_{\lambda,\lambda'}(P,x,\mathbf{k}_T,\Delta,n)$$

$\xi = 0$

$\mathcal{FT}: \Delta_T \Longleftrightarrow \vec{b}_T$

$$W^{[\Gamma]}_{\lambda,\lambda'}(P,x,\mathbf{k}_T,0,n) = \Phi(x,\mathbf{k}_T)$$

$$\mathcal{F}_{\lambda,\lambda'}(x,\vec{b}) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{i\vec{\Delta}_T\cdot\vec{b}} F_{\lambda,\lambda'}(x,0,\vec{\Delta}_T)$$

TMD

Impact-GPD

Unifying Transverse Structure of Nucleon GTMDs

GTMD--Meissner Metz Schlegel 07, 08

$$W^{[\Gamma]}_{\lambda,\lambda'}(P,x,\mathbf{k}_T,\Delta,n) = \int dk^- \, W^{[\Gamma]}_{\lambda,\lambda'}(P,k,\Delta,n)$$

Integ. small component !!!

$$\mathcal{FT} : \Delta \Longleftrightarrow \vec{b}$$

$$W^{[\Gamma]}_{\lambda,\lambda'}(P,k,\Delta;n) \Longleftrightarrow W^{[\Gamma]}_{\lambda,\lambda'}(P,k,\vec{b};n)$$

Wigner functions--Belitsky Ji Yuan, 04

Reduce to TMDs, GPDs, Impact GPDs
Relations among them?